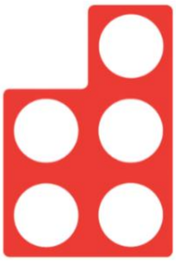
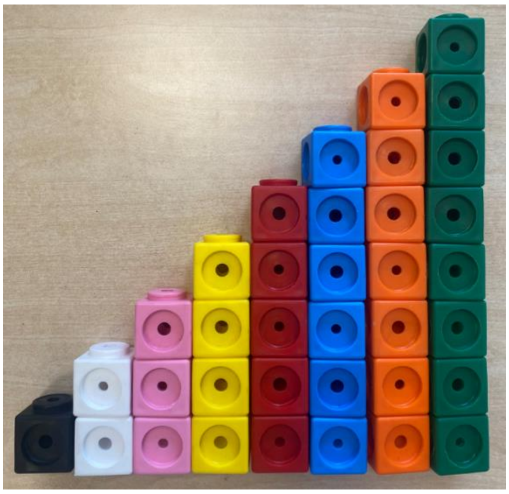
Planning Overview
Year 6 Ratio and Proportion

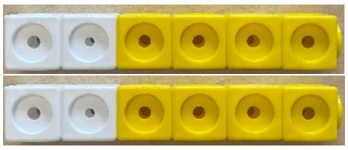
Pupils should be taught to:

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- Solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

AS/MD–1 Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).
6AS/MD–3 Solve problems involving ratio relationships

Describing the proportional relationship between 2 factors using ratio and proportion	Using numicon ask children to describe the colour of the tiles and the number of holes. For every 1 red tile we have 5 holes.  Ask children what would happen if we then had more than one tile. For every 1 red tile we have 5 holes. For every 2 red tiles we have 10 holes. For every 3 red tiles we have 15 holes. Ask children how many red tiles they would have if they had 25 holes? Extend to using the language of 'for every' to compare 2 different objects e.g. compare multilink towers.  Using this image can the children create some 'for every' sentences. E.g. For every 2 white cubes there are 4 yellow cubes. 

What if we could see 4 white cubes, how many yellow cubes would we see then?



2 white and 4 yellow = 6 cubes altogether.
4 white and 8 yellow = 12 cubes altogether.
Why has the total number of cubes increased by 6 in the second sequence.

Children could continue the sequence in a table format e.g.

White Cubes	Yellow cubes
2	4
4	8
?	12
8	?

What if we could see a total of 30 white and yellow cubes, how many of each colour would we be able to see?

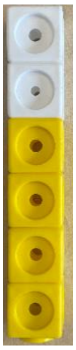
If we were comparing the pink and red cubes, how much would the total increase by each time? Why?

Ensure that the vocabulary for RaTiO – TO every is included on your working wall.

Link to Proportion

In this plan, proportion has been covered alongside ratio, you may prefer to teach proportion after you have taught ratio.

Introduce the word Proportion and the difference between Proportion and Ratio
ProportIoN – IN every
RaTiO – TO every



Using the same resource as when we introduced Ratio, write a range of proportion statements e.g. when comparing white to yellow cubes 2 in every 6 are white, 4 in every 6 are yellow.

Written as a fraction this would be $\frac{2}{6}$ are white, $\frac{4}{6}$ are yellow.

Create a diagram of your group, can you create Ratio and Proportion statements? e.g. ratio of dark to light hair 2:3

Solve simple ratio problems

Allow children to consolidate their understanding of ratio using some of the problems below.

e.g. Vase Problem – NCETM PD materials

What can the children tell you about the number of vases and the number of flowers?

For every ______ vase I have _______ flowers.

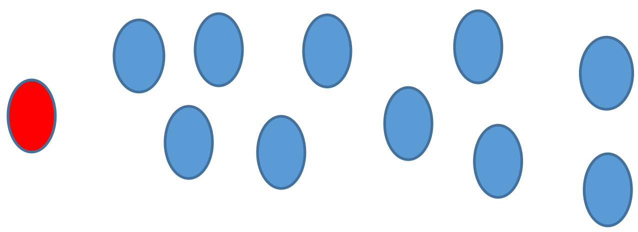
If there were 3 vases how many flowers would there be?

We can ask children to think about the number of vases if we know the number of flowers.

If I can see 20 flowers, then how many vases can I see? 20 flowers divided by 5 (because there are 5 flowers for every one vase) means we must have 4 vases.

For every 4 vases there are 20 flowers.
Allow children time to explore.

'For every one red jellybean there are 10 blue jelly beans'



Can children create a table to show the relationships?

Red jelly beans	Blue jelly beans
1	10
2	??
?	30

What statements can the children make using this stem sentence?

For every _______ red jelly bean there are _____ blue jelly beans.
If I can see ____ red jelly beans then I can see ______ blue jelly beans.
If I can see _______ blue jelly beans then I can see ________ red jelly beans.

Red jelly beans are in a ratio of 1:10 with blue jelly beans.

To apply this to another problem we could say for every 3 blue flowers we have 5 red – we have blue to red flowers in a ratio of3:5
We can also show this as a table

Blue flowers	Red flowers
3	5
6	10
9	15
?	20

We can also add an additional row to our table and start to think about what the total number of flowers are.

Blue flowers	Red flowers	Total flowers
3	5	8
6	10	16
9	15	?
?	20	32

Encourage children to think about what times tables children can see in the table.

A table like this helps us to talk about the ratio and the proportional relationships

The flowers have a ratio of 3:5 blue to red.
In every 8 flowers there are 3 blue flowers and 5 red flowers (proportion)
$\frac{3}{8}$ are blue flowers, $\frac{5}{8}$ are red flowers.

Look at this NCETM problem with the children

Litres of petrol	1	2	3	4	5	6	7	8	9	10
Miles driven	7	14	21	28	35	42	49	56	63	70

ncetm.org.uk

- How far can you drive for every 1 litre of petrol?

  For every 1 litre of petrol, you can drive 7 miles.
- How can this help us to find how many miles can be driven for 2 litres of petrol?
- How many litres of petrol would be needed t travel 21 miles? How do you know?
- Complete the remaining unknown values.
- How far could you drive with 40 litres of petrol? How many litres do you need to drive 175 miles?

Using a bar model to tackle ratio problems where we know the whole and the ratio

Tell the children that in a class there are 30 children at a ratio of 2 boys to 1 girl (2:1)

boy	boy	girl

This bar model has 3 parts altogether. If we take our 30 children and think about sharing this equally across our 3 parts – each part would get 10 children

Boy 10	Boy 10	Girl 10
Boys 20		Girls 10

Mastery

Sam and Tom share 45 marbles in the ratio 2:3.

How many more marbles does Tom have than Sam?

Look at this problem in the same way. How many parts are there altogether?
What is the whole?
what will each part be worth when we share the whole between them?
How many marbles will Sam get?
How many will Tom get?
What is the difference between the number of marbles that the boys have?

Mastery with Greater Depth

Jim and Harry share some marbles in the ratio of 3:5. Jim gets 24 more marbles than Harry. How many marbles do they share altogether?

This problem is harder in that the children are required to think about the ratio and the difference initially.

We need to think that Jim gets 2 extra parts than Harry. Those 2 parts totals are 24. So we now know that one part is 12.

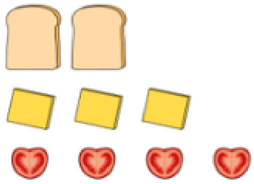
We can populate the rest of the parts in the bar model now. Work out Jim's total, Harry's total and their combined total.

h	h	h	j	j	j	j	j
12	12	12	12	12	12	12	12
36			60				

Use ratio and proportion to solve problems with 3 unknowns

Show children this problem from NCETM

To make a cheese and tomato sandwich we need
2 slices of bread, 3 slices of cheese and 4 slices of tomato.



Ask the children to tell you what the ratios are involved in this problem.
They should be able to tell you it's 2:3:4
Can they draw a bar model to show this ratio?
Can they transfer this data to a table?

Number of sandwiches	bread	cheese	Tomato
1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

Can children see what happens to this proportional table to scale up one sandwich to become 12 sandwiches?

Number of sandwiches	bread	cheese	Tomato
1	2	3	4
12	24	36	48

We have made our one sandwich 12 times bigger so we need to proportionally make all of the amounts of our ingredient 12 times bigger too.

Can they complete a similar chart to think about the ingredients involved in making 8 sandwiches

Number of sandwiches	bread	cheese	Tomato
1	2	3	4
8			

Can children use the same reasoning and a table to work out how many sandwiches we can make with 24 slices of cheese?

Number of sandwiches	bread	cheese	Tomato
1	2	3	4

		24	

Encourage children to think about how we have scaled 3 up to make it to 24. We have multiplied it by 8. We therefore proportionally need to scale up the other ingredients 8 times.

What if 16 tomatoes were used. Can children work out how many sandwiches have been made now just using their understanding of multiplication?

Children to draw a table or use their understanding of multiplication to support their understanding to solve the following SATs problems

19 David and his friends prepare a picnic.

Each person at the picnic will get:

3 sandwiches

2 bananas

1 packet of crisps

The children pack **45** sandwiches.

How many **bananas** do they pack?

bananas

19

Sapna makes a fruit salad using bananas, oranges and apples.

For every one banana, she uses 2 oranges and 3 apples.

Sapna uses 24 fruits.

How many **oranges** does she use?

oranges

Mastery

To make a tomato pizza topping for a normal pizza, Jake uses 300 g of tomatoes, 120 g of onions and 75 g of mushrooms.

Jake wants enough sauce for a giant pizza, so he uses 900 g of tomatoes.

What mass of onions will be used?

How many 120 g boxes of mushrooms will he have to buy?

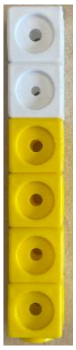
Mastery with Greater Depth

Jake has now made his giant pizza. He says, 'I made three times as much sauce to cover the giant pizza as I do to cover a normal pizza, so the giant pizza is three times as big as the normal pizza.'

Do you agree with Jake?

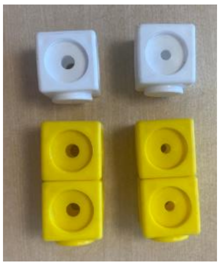
Simplifying ratio to solve proportion problems

Children will have simplified fractions during the fractions unit of work so may be able to quickly apply their understanding to simplifying ratio and proportion. Return to the images/multilink towers shown at the beginning of the unit.



We used 6 multilink
The ratio of white to yellow is 2:6
The proportion of white cubes is 2 in every 6
The propotion of yellow cubes is 4 in every 6
$\frac{2}{6}$ of the cubes are white

$\frac{4}{6}$ of the cubes are yellow

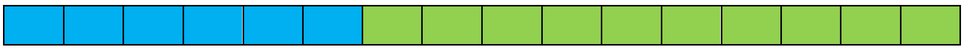


Explain to children that in the same way that we can simplify fractions we can simplify ratio and proportion.

If we split the image up to look like this, can children see that the proportion of white cubes is 1 in every 3?

Can children create the remaining ratio, proportion and fraction statements about these cubes?

Repeat with another image.



Ask children to identify what this ratio is.
'The ratio of blue to green is 6:10'

If we took our above bar and made every 2 sections into 1 section, we would create a bar that looks like this.

We have altered each part of the bar model in the same way so that we maintained the proportions of our original bar.

What ratio have we simplified 6:10 down to?

'The ratio that we have simplified it down to is 3:5'

Ask the children what the relationship is between both of those ratios.

6 and 10 are both multiples of 2. Because if this we can divide both of parts of this ratio by 2 to create a simplified ratio of 3:5.

Ask children to practice simplifying other ratios
6:12
10:15
20:24
150:600

Now look at a recipe problem where the proportions involved give us large ratios

19 This is Kirsty's recipe for breakfast cereal.

50 grams of oats
30 grams of raisins
40 grams of nuts

If she uses 125 grams of oats, how many grams of raisins does she need?

g

In this recipe we need to look at what the ratio of oats to raisins is and scale this down. We need to do that because it is the only way to understand the relationship between 125g of oats and the mystery number of raisins.

Oats to raisins in the recipe is 50:30 we can simplify this down to 5:3. We can then apply this to 125g of oats.

We need to think about how we can scale up 5 to make it 125. To do this we multiply it by 25.

To maintain the proportions of the ratio we need to multiply the other part of our ratio, which is 3, by 25 as well.

We can now state that under the ration 5:3 if we had 125g oats we would have 75g of raisins.

Why does this work?
'We took the ratio 50:30 and we simplified this using the common multiple of 10 to 5:3. We then thought about how we would need to scale up 5 to get it to be 125 and then scaled the other side of the ratio up in the same way. 125:75 is the same proportions as 50:30 and 5:3'

Ask children to tackle the below problem where they need to simplify the ratio of 400:125 and use this simplified ratio to work out and unknown amount of chocolate when given a quantity of cream.

18 Here are the ingredients for chocolate ice cream.

cream	400 ml
milk	500 ml
egg yolks	4
chocolate	120 g
sugar	100g

Stefan has only 300ml of cream to make chocolate ice cream.

How much **chocolate** should he use?

[ ] g

2 marks

Using and applying ratio and proportion to solve a range of problems

Ask children to apply their understanding of ratio and proportion, bar models, ratio grids or multiplication and division to tackle the following problems.

22 Here is a recipe for fruit smoothies.

Recipe

10 strawberries

$\frac{1}{2}$ litre of orange juice

250ml yogurt

1 banana

Makes two smoothies

Stefan uses the recipe to make smoothies.
He uses 1 litre of yogurt.

How many strawberries does he use?

[ ]

22a

1 mark

Amir uses the same recipe.

He wants to make 5 smoothies.
He has 1 litre of orange juice.

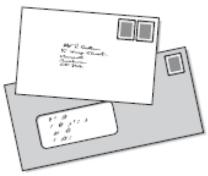
How many **more** millilitres of orange juice does he need?

[ ] ml

2 marks

1 smoothie				
2 smoothies	10 strawberries	½ litre orange juice	250ml yoghurt	1 banana
			1 litre yoghurt	

Two letters have a total weight of **120 grams.**



One letter weighs **twice as much** as the other.

Write the weight of the **heavier** letter.

[g]

1 mark

Can children identify that $\frac{2}{3}$ of the total weight will be related to the heavier letter so they need to find $\frac{2}{3}$ of 120g to find the answer?

Mastery
You can buy 3 pots of banana yoghurt for £2·40. How much will it cost to buy 12 pots of banana yoghurt? A child's bus ticket costs £3·70 and an adult bus ticket costs twice as much. How much does an adult bus ticket cost? To make a sponge cake, I need six times as much flour as I do when I'm making a fairy cake. If a sponge cake needs 270 g of flour, how much does a fairy cake need?

Solving problems involving scaling	Look at Cuisenaire rods and find pairs of rods that satisfy a range of sentences. Find a rod that is twice the size of another rod Find a rod that is three times the size of another rod Find a rod that is half the size of another rod Apply that understanding through the use of bar models to solve scaling problems

If Sam and Sarah shared £36 but Sam got twice as much as Sarah how much would each person get?

Sarah	
Sam	Sam

= £36

We can see 3 parts but we need to make sure that Sam gets 2 of those parts and Sarah only gets 1.

£36 into 3 parts makes each part £12
Sam gets 2 parts =£24
Sarah gest 1 part = £12

We can apply this thinking to problems like this

20 Chen is cooking some pasta.

The recipe says he needs 350 grams of pasta for 4 people.

How many **kilograms** of pasta does he need for **12 people**?

kg

We know how much pasta we have for 4 people

350g
4 people

If we scaled up the number of people to become 12 we end up with 3 times that many sections of the bar. To keep our bars proportional, we also need to have 3 times as many sections of the top bar

350g	350g	350g
4 people	4 people	4 people

Now we have 4 people and 1050g of pasta.

We could also think about scaling this problem down to scale it back up again. If we needed to know how much pasta we needed for 10 people for instance.

We would need to know how much pasta each person would need we would need to divide the 4 people by 4 to create one person and the 350g by 4 to find out how much pasta that one person will eat.

350g			
4 people			
1 person 87.5g	1 person 87.5g	1 person 87.5g	1 person 87.5g

So now that we know each person will eat 87.5g of pasta we can use that to find out what 10 people will eat by multiplying that by 10.

10 people will need 875g of pasta.

Mastery

Sam has 9 fewer sweets than Sarah. They have 35 sweets altogether.

How many sweets does Sam have?

Here we have 2 parts but are told that Sarah has 9 more than Sam

Firstly, we need to take 9 away from 35 and give that to Sarah

Then we divide the remaining 26 between both Sam and Sarah giving them 13 each. We need to remember to add Sarah's extra 9 onto her total.

Sam has 13 and Sarah has 22

As a bar model this could look like this

35		
Sam	Sarah	
26		9
13	13	9

Use multiplication to solve correspondence problems

Allow children to explore how many combinations of outfits they can make with one jumper and one pair of shorts, one jumper and 2 pairs of shorts, one jumper and 3 pairs of shorts. Record in a table like this;

Jumper	Shorts	Combinations
1	1	1
1	2	2
1	3	3

Now add in another jumper. So how many combinations can the children make with 2 pairs of shorts and 2 jumpers? 2 pairs of shorts and 3 jumpers? 2 pairs of shorts and 4 jumpers?

Jumper	Shorts	Combinations
2	2	4
2	3	6
2	4	8

Are the children starting to notice a pattern between the number of variable and the number of combinations?

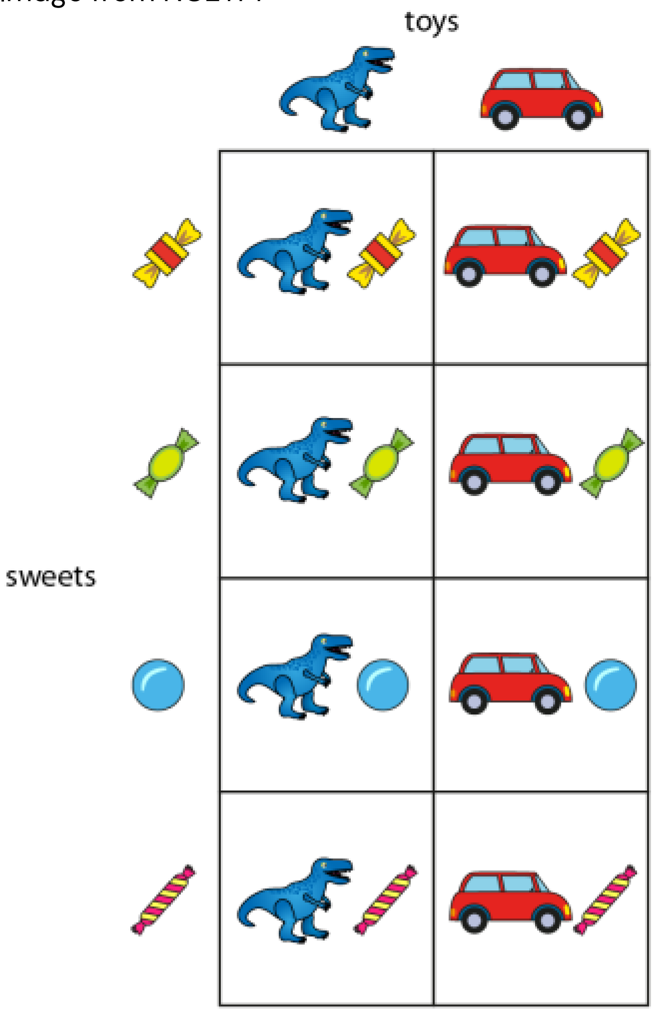
Can they make a prediction as to how many combinations they can make using these variables?

Image from NCETM

3 hats x 2 coats = 6 combinations

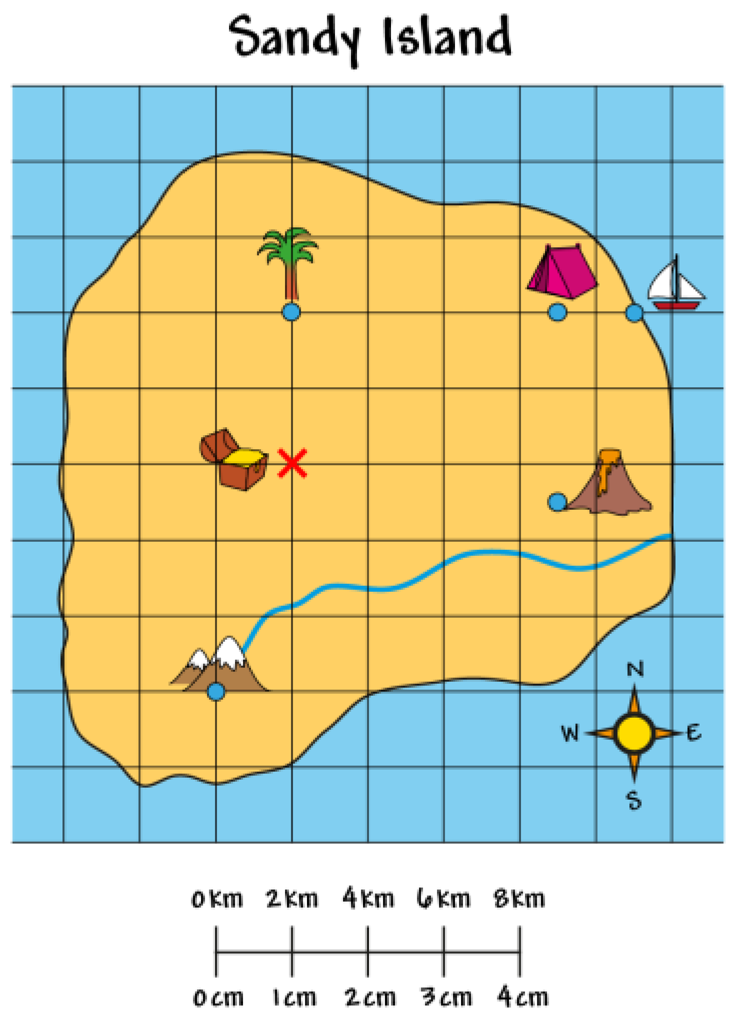
Can children explain how many different combinations of presents they could make with 2 different toys and 4 different types of sweets?

Number of toys x number of sweets = combinations of presents
Image from NCETM



Children to solve correspondence problems such as these from the NCETM PD materials

- *'Rajesh takes seven different pairs of socks and three different pairs of shoes on holiday with him. How many different combinations of shoes and socks does he have?'*
- *'Before a football game, each person on team A shakes hands with each person on team B. How many hand-shakes are there if there are:*
  - *five people in each team?*
  - *eleven people in each team?'*

	• *'Frankie has some different pairs of trousers and six different T-shirts. He can make twenty-four different outfits. How many pairs of trousers does Frankie have?'* • *'I have two boxes, each containing a set of different toys. There is only one of each type of toy. I draw one toy out of each box to make a pair. If I can make twenty-four different pairs of toys, how many toys were in each box?'*
Scale factors	Ask children to look at a simple map such as this one from the NCETM.  What do children think the scale below the map is indicating? How far away is the palm tree from the treasure? Its 2cm on the drawing but what does the scale tell you this actually is?

How would we represent a distance of 1km on the map?
Allow children to explore a range of maps with a range of scales.

If necessary, children can create themselves a table to help them to scale distances

1 cm	50m
2cm	100m
3cm	150m
4cm	200m

Provide children with squared paper and a given scale.
1cm = 3km

Ask them to draw a map and represent certain landmarks on their maps with related distances.

'The park needs to be 15km away from the beach'

Ask them to add some landmarks of their own and to write some questions regarding these at the bottom of their maps for their peers to answer.

Children to apply this skill to other contexts

'1 cm on this drawing represents 30cm in real life. Fill in the real-life measurements of the car.'

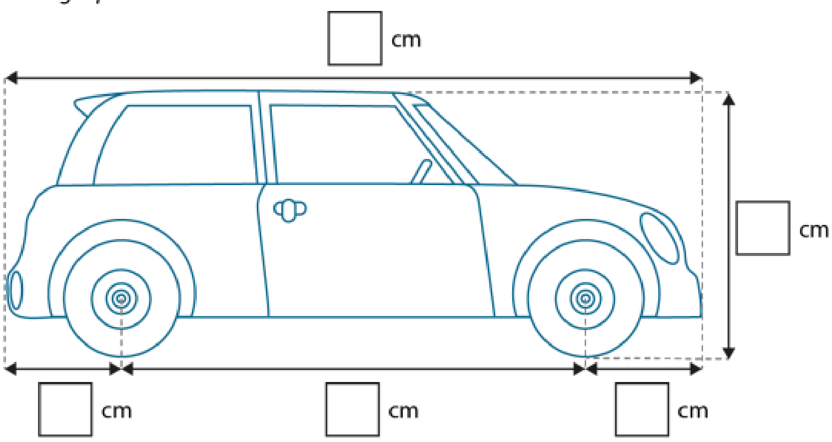


Image taken from NCETM PD materials

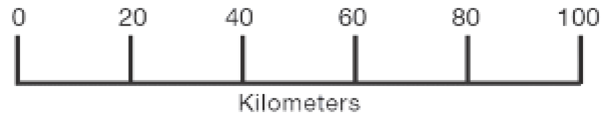
Ask children to apply this understanding to answer questions in a different context like this from Teaching for Mastery

1. For every 1 litre of petrol, Miss Smith's car can travel about 7km.
   a. How many kilometres can Miss Smith's car travel on 6 litres of petrol?
   b. Miss Smith lives about 28km from school. How many litres of petrol does she use to get to school?

Apply to SATs questions

25 The distance between the two cities is **350km.**

On a map, **1cm** represents **20km.**



What is the **distance** between the two cities **on the map?**

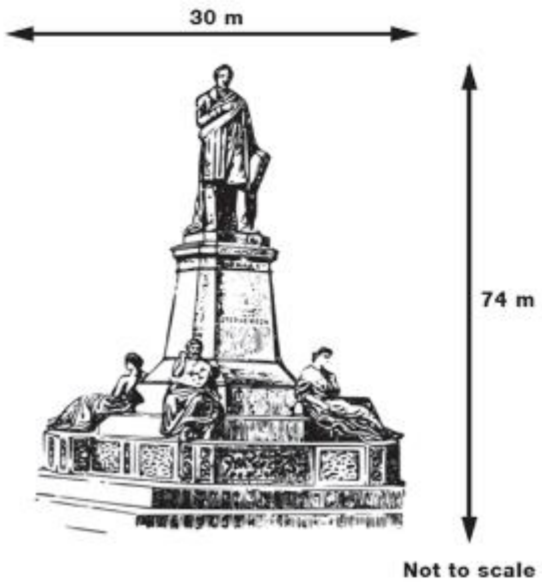
Show your method

cm

2 marks

23 The Stephenson monument is a large statue in Newcastle.

It is 30 meters wide and 74 meters tall.



Sia makes a scale model of the Stephenson monument.

Her model is 37 centimeters tall.

How **wide** is her model?

cm

1 mark

Scale factors and shape

Show children this image and ask them to use proportional language to talk about the lengths of the squares

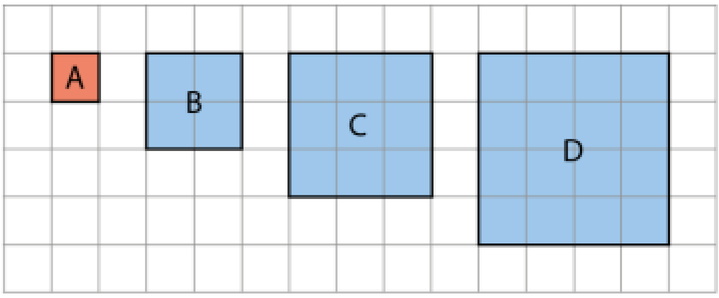


Image taken from NCETM PD materials

Square b has sides twice the length of square a
We can write this as side length b = side length of a x 2
Or
Side length of a = side length of b x ½

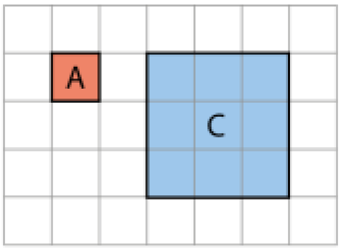


Image taken from NCETM PD materials

How can we describe the scale factor of these 2 shapes?

Side length c =
Side length a =

Ask children to apply this understanding to be able to draw shapes when given the scale factor.

Scale the length of the sides by a scale factor of 2

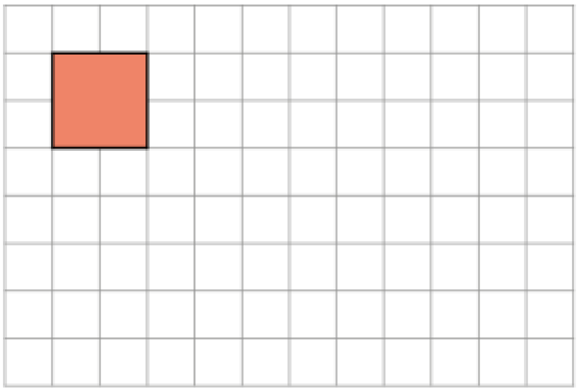
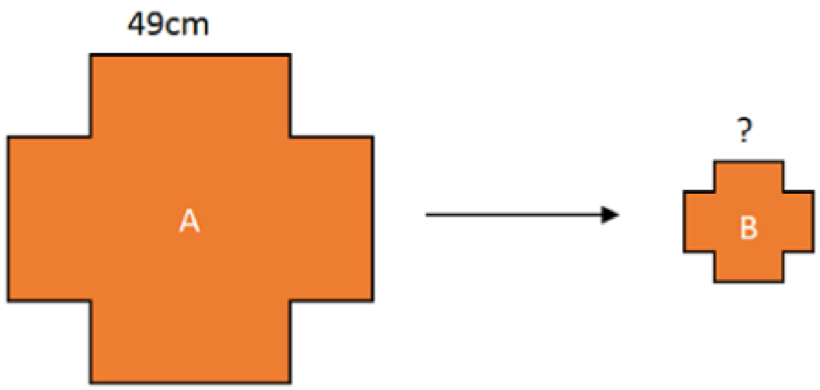
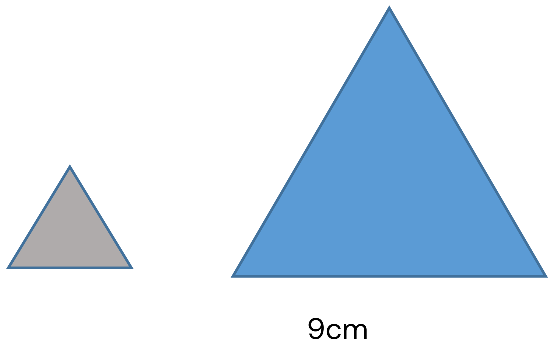


Image taken from NCETM PD materials

	Relate this understanding of scale factors to other 2D shapes. Work out the original length of the side after it has been enlarged by a scale factor of 7.  Image taken from BBC bitesize Apply to larger problems involving other elements of shape and measure  The blue triangle has been enlarged by a scale factor of 3. What is the perimter of the white triangle?