Planning Overview
Year 6 Multiplication and Division

Perform mental calculations, including with mixed operations and large numbers
Identify common factors, common multiples and prime numbers
Use their knowledge of the order of operations to carry out calculations involving the four operations
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
Solve problems involving addition, subtraction, multiplication and division
Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

6AS/MD–1 Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number)
6AS/MD–2 Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.

	Teaching and Learning
Introduction	'What is multiplication?' How do children answer this question? Are they able to recall all elements of multiplication that they have covered in previous year groups? Add to the working wall and let children know that they will be linking together a range of elements of multiplication and applying them to worded and more substantial problems during this unit of work. Include appropriately pitched SATs questions throughout to develop children's confidence.
Common multiples and common factors	Revisit factors and multiples with the children and ask them to share examples of both with their partner. Consolidate their understanding by playing the factors and multiples game. Encourage children to discuss how knowledge of common factors and common multiples may help them to win the game.

	https://nrich.maths.org/factorsandmultiples

Factors and Multiples Game

Age 7 to 16
Challenge Level ★

This is a game for two players.

The first player chooses a positive even number that is less than 50, and crosses it out on the grid.

The second player chooses a number to cross out. The number must be a factor or multiple of the first number.

Players continue to take it in turns to cross out numbers, at each stage choosing a number that is a factor or multiple of the number just crossed out by the other player.

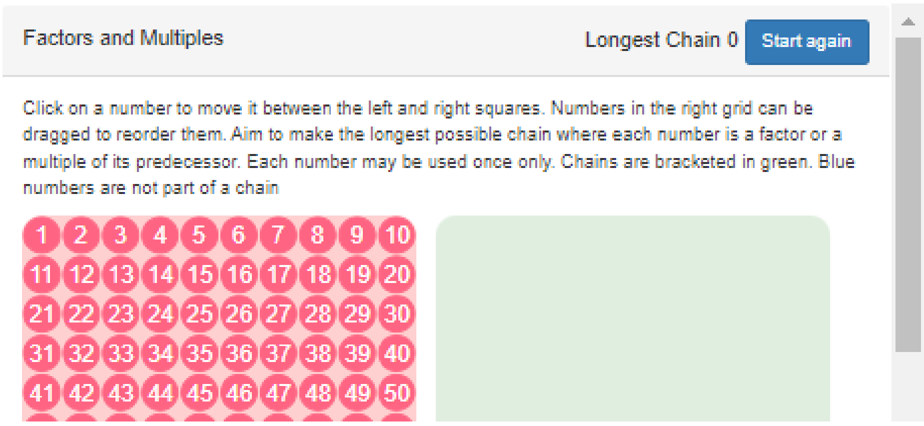
The first person who is unable to cross out a number loses.

Play a few times to get a feel for the game.

Do you have any winning strategies?

Here is an interactive version of the game in which you drag the numbers from the left hand grid and drop them on the right hand grid. Alternatively, click on a number in the left hand grid and it will transport to the earliest empty location in the right hand grid. You can rearrange the numbers in the right hand grid by dragging and dropping them in position. The integer in the top right hand corner grows with the number of factors/multiples you have in a row.

Tablet version Install in home page



Multiples

Ensure children have time to practise finding common multiples to check understanding from Year 5.

Apply to a range of word problems including answering SATs style questions. e.g.
Write all the common multiples of 3 and 8 that are less than 50.

Amir says,

'All numbers that end in a 4 are multiples of 4'.

Is he correct?

Circle **Yes** or **No**.

Yes / No

Explain how you know.

Here is a sorting diagram with four sections, **A**, **B**, **C** and **D**.

	multiple of 10	not a multiple of 10
multiple of 20	**A**	**B**
not a multiple of 20	**C**	**D**

a) Write a number that could go in section **C.**
b) Section **B** can never have any numbers in it. Explain why.

364 is a multiple of 7 but not a multiple of 3. 384 is a multiple of 3 but not a multiple of 7. Find a number between 364 and 384 that is both a multiple of 7 and a multiple of 3.

Factors
Ensure children have time to practise finding common factors to check understanding from Year 5. Apply to a range of word problems including answering SATs style questions.

Find 3 factors of 36 that are not factors of 12.

Write three factors of 30 that are **not** factors of 15

Is 1 always, sometimes or never a common factor of 2 numbers?

Children to understand the phrase Highest Common Factor (HCF). HCF is found by finding all common factors of two numbers and selecting the largest one.

E.g. 8 and 12 have the common factors 1, 2 and 4. 4 is the Highest Common Factor.
Can children find the HCF of a selection of 2 numbers.

Tom and Ellie think of 2 numbers. These are common factors of their numbers: 1, 3, 5 and 15. What could their numbers be? Find a rule that could explain every possibility.

Here are three digit cards

| 1 | 5 | 6 |

Choose two cards each time to make the following two-digit numbers.

The first one is done for you.

an even number	5	6
an prime number		
a common factor of 60 and 90		
a common multiple of 5 and 13		

The factors of 11 sum to 12. Write the other number whose factors sum to 12.

Children can then apply their knowledge to solve the NRICH problems below.

Abundant Numbers

Age 7 to 11
Challenge Level ★

To find the **factors** of a number, you have to find **all** the pairs of numbers that multiply together to give that number.

The factors of 48 are:

1 and 48

2 and 24

3 and 16

4 and 12

6 and 8

If we leave out the number we started with, 48, and add all the other factors, we get 76:

$1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 = 76$

So .... 48 is called an **abundant** number because it is less than the sum of its factors (without itself). (48 is less than 76.)

See if you can find some more abundant numbers!

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<td>

Multiplication Squares

Age 7 to 11
Challenge Level ★

In the 2×2 multiplication square below, the boxes at the end of each row and the foot of each column give the result of multiplying the two numbers in that row or column.

<table>
<tr><td>7</td><td>5</td><td>35</td></tr>
<tr><td>3</td><td>4</td><td>12</td></tr>
<tr><td>21</td><td>20</td><td></td></tr>
</table>

The 3×3 multiplication square below works in the same way. The boxes at the end of each row and the foot of each column give the result of multiplying the three numbers in that row or column.

<table>
<tr><td></td><td></td><td></td><td>15</td></tr>
<tr><td></td><td></td><td></td><td>108</td></tr>
<tr><td></td><td></td><td></td><td>224</td></tr>
<tr><td>144</td><td>8</td><td>315</td><td></td></tr>
</table>

The numbers $1 - 9$ may be used once and once only.

Can you work out the arrangement of the digits in the square so that the given products are correct?

</td>
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Prime numbers

</td>
<td>

Have the children retained their ability to recall Prime Numbers to 19 from Year 5? Ask the children to apply their knowledge of prime numbers to solve the following problems.

How many factors does a prime number have?

38 doesn't appear in any times tables so it must be prime. Agree or disagree? Explain.

Is it sometimes, always or never true that a number ending in a 7 is a prime number? Explain.

Emma thinks of two prime numbers. She adds the two numbers together. Her answer is 36. Write all the possible pairs of prime numbers Emma could be thinking of.

Is 1 a prime number? Explain.

Provide children with examples of SATs style questions to apply their knowledge.

Circle the **prime** number.

95 89 87

Explain how you know the other numbers are **not** prime.

Chen chooses a prime number. He multiplies it by 10 and then rounds it to the nearest hundred. His answer is 400. Write **all** the possible prime numbers Chen could have chosen.

</td>
</tr>
</table>

Here are five numbers.

~~2~~ 3 4 5 6

Write each number on the correct cards.

The number 2 has been written on the correct cards for you.

Prime numbers	Factors of 12	Factors of 15
2	2	

Allow children to progress on to exploring the NRICH problem below.

Code Breaker

Age 7 to 11
Challenge Level ★★

Many modern codes are based on two very large prime numbers multiplied together.

This problem is based on a code using two different prime numbers less than 10. These two primes have been multiplied together and the resulting number has been used to shift the alphabet forward to new letters, assuming that A is at position 1, B at position 2 etc. For example, if the two prime numbers were 2 and 3, then to make the code, the alphabet would be shifted forward by 6 places. A would become G, B shifts to H and so on.

Which way will you need to shift the letters to decode?

When you have deciphered the code, there will be one word which will remain coded. You can decipher this word by adding the two prime numbers together and shifting the letters again.

Can you find the doubly coded word in this sentence?

JZF SLGP FUDFNHG TE

Square and cube numbers

Ensure the children can recall what square and cube numbers are. Complete the table below applying your knowledge of square numbers.

	5 x 5	
7^2		
3^2		
	4 x 4	
		64
		1

Complete the table below applying your knowledge of cube numbers.

3^3		
5^3		
	6 x 6 x 6	
		216
	4 x 4 x 4	
		8

What do you know about the factors of square and cube numbers?

Can the children apply their knowledge to solve SATs style questions.

e.g.

Put these values in order with the smallest first.

5^2 3^2 3^3 2^3

☐ ☐ ☐ ☐

smallest largest

A **square** number and a **prime** number have a total of 22.
What are the two numbers?

☐ + ☐ = 22

square number prime number

Extend to problems such as:

Josh says, “I am thinking of 2 numbers. When I add them I get a prime number. When I multiply them I get a square number.”

Mina says, "I am thinking of 2 numbers. When I add them I get a square number. When I multiply them I get a prime number."

What are the two numbers they are thinking of?

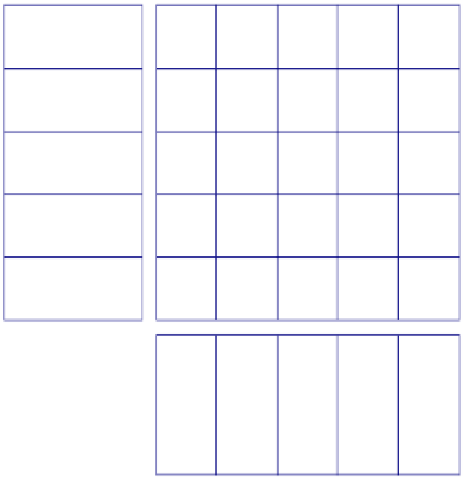
Dexter works out 20 squared. Annie works out 20 cubed. Find the difference between Dexter's and Annie's numbers.

Encourage children to apply their knowledge of what they have learnt to the following problem from NRICH.

Factors and Multiples Puzzle

Age 11 to 14 ★

To try this puzzle you will need a copy of the playing board and cards. You can download a copy to print.



Note: Children will see the card 'Triangular Numbers' within this activity. Decide when and how you will introduce triangular numbers to your children.

1. Cut out the 10 heading cards and put one in each of the 10 spaces round the playing board.

PRIME NUMBERS	TRIANGULAR NUMBERS
SQUARE NUMBERS	FACTORS OF 60
NUMBERS LESS THAN 20	MULTIPLES OF 3
NUMBERS MORE THAN 20	MULTIPLES OF 5
ODD NUMBERS	EVEN NUMBERS

2. Cut out the 25 number cards and place each one in a different square on the playing board so that the number satisfies the condition given by the heading card for that row *and* the condition given by the heading card for that column.

1	2	3	4	5
6	7	9	10	11
12	15	16	18	20
21	23	24	25	30

Mental methods for multiplication

Children need to be familiar with a range of mental strategies and be able to recognise when to use each one according to the type rather than size of the numbers involved. A range of numbers of different sizes should be used to avoid children automatically using a written method for larger or decimal numbers when a mental method may be more appropriate.

Give the children the following set of calculations.

12 x 11 =	3 x 6 x 5 =
146 x 33 =	19 x 8 =
3,745 ÷ 5 =	60 x 50 =
87 x 7 =	336 ÷ 6 =
21 x 4 =	16 x 1,000 =
16 x 4 =	40 x 60 =
180 ÷ 3 =	3,200 ÷ 100 =

How would they solve each of these calculations? Choose the most appropriate strategy and show your workings:

- Known facts
- Reordering
- Partitioning
- Rounding and adjusting
- Formal written method
- Place value
- Double one side and halve the other

Ask the children to visit other children to see if they have sorted the questions in a different way. They must then try to convince them why they are correct. Encourage the children to go back and add the alternative thought to their work.

Play pointless:

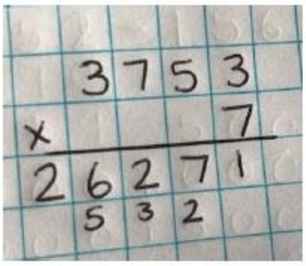
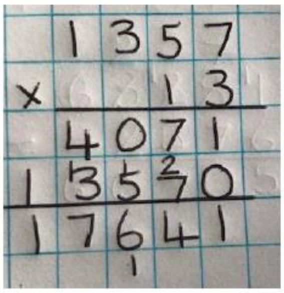
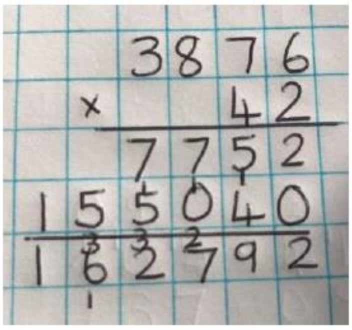
Has anyone used a strategy that isn't listed?

Can they add another calculation to each list?

As a class, has anyone come up with an answer no one else has?

	Spot the mistake. Provide the children with calculations where a common misconception has occurred. Can the children find the mistake? e.g Emma says that if she knows 5 x 4 =20, then she can work out 50 x 40 = 200. What mistake has she made? 5 x 7 = 35 Ben says he can use this fact to work out 0.5 x 7. Do you agree or disagree? What other answers can he work out? Is it always, sometimes or never true that when I multiply by a multiple of 10, I add a zero on the end? Kate and Lucy work out the answer to this calculation in two different ways… Solve 24 x 15 - Kate uses partitioning 24 x 10 = 240 24 x 5 = 120 240+120 = 360 - Lucy uses doubling and halving: 24 X 15 12 x 30 = 360 Whose method is most efficient and why? Can you suggest any other ways you could have done the calculation? When children are confident with these methods, they can move on to learning about factorising. Finding factors of numbers (factorising) is a useful mathematical skill that can help pupils do mental multiplication and division calculations. They may also find it helpful to understand the **commutative law** for multiplication. You can start with a question with small numbers to illustrate the process: 12 × 8 is the same as 6 × 2 × 8, so I can do 6 × 8 = 48 first and then × 2 (double my answer to 96. With multiplication, you may do the multiplication in any order. I started the calculation by factorising 12 into more manageable numbers.

	120 ÷ 15 is the same as 120 ÷ 5 = 24 and then 24 ÷ 3 = 8 (We have divided by 5 and then by 3 because 5 × 3 = 15) This method can then also be extended to larger numbers for both multiplication and division questions. 288 ÷ 12 First think of factors of 12 (12 = 2 × 2 × 3) So 288 ÷ 12 is the same as 288 ÷ 2 = 144, then 144 ÷ 2 = 72, then 72 ÷ 3 = 24. So 288 ÷ 12 = 24 325 x 6 (First think of factors of 6 (2 × 3) So 325 × 3 = 975, then 975 × 2 = 1950
Estimating multiplication questions.	Remind children why it might be good to estimate an answer before you tackle a calculation. Provide the children with a calculation e.g. 3,456 x 72 = What would be a good estimation for the answer? How could we use rounding to help us? What would the best numbers be to round to? Model rounding 39,456 to the nearest 1,000 and 72 to the nearest ten. 3,000 x 70 = What is the actual answer? Is your estimation near? Can it help you check you have the right answer? What about if we round 3,456 to the nearest 100? Will the estimation be more precise? However, is the calculation easy to do mentally? Therefore, which would be the best estimation? Provide children with examples of questions to estimate the answers to. 2,564 x 9 = 3,452 x 31 = 29,450 x 87 = Discuss with the children how context would change estimations. What would happen if we were ordering flooring for a room? Would this be the best estimate? Why not? Explain to the children that when we rounded, we rounded down so we would not have enough carpet. What would have been a better estimate? Sam has completed these calculations. How would rounding have helped him to know that he has made an error? 4563 x 56 = 50193 6839 x 5 = 30424

	Some children are asked to work out 308 x 19. a) Which is the best estimate to use to check their answers? Why? 300 x 10 300 x 20 310 x 20 300 x 19 b) What is the answer? c) How far off was your estimation? Estimation can now be consolidated as the children recap written strategies with larger numbers. Ensure that the children estimate the answer to each question before solving them.
Written methods of multiplication	Recap Y5 learning. Ensure that children can confidently complete written multiplication calculations using the formal methods of short and long multiplication with a different number of digits in the numbers. e.g.   Ensure they can multiply larger two-digit numbers by four-digit numbers before moving on. e.g.  Choose a 4-digit target number. Roll the dice 4 times to generate 4 digits. Use them to make two 2-digit numbers. Play against a partner. Who can get the closest product to the target number?

Allow children opportunity to apply their knowledge to worded problems that involve multiplication, highlighting the mathematical vocabulary relating to multiplication.

e.g.

A printer can print 95 documents in a minute. How many documents can it print in three quarters of an hour?

Jamie does 56 sit ups every day. How many sit ups will he do in a year?

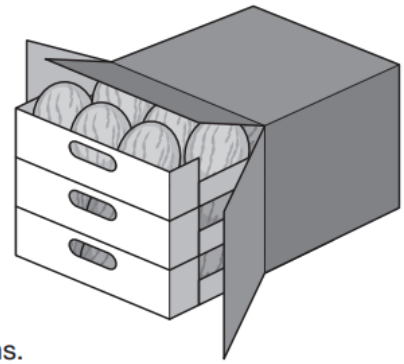
One side of a rectangle is 36cm and the other is 0.82m. What is the area of the rectangle?

Sample SATS question

A box contains trays of melons.

There are 15 melons in a tray.

There are 3 trays in a box.



A supermarket sells **40** boxes of melons.

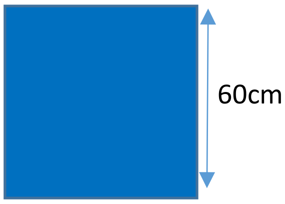
How many melons does the supermarket sell?

Allow children to complete missing digit problems.

```
   ? 1
 x 2 ?
 -----
  324
 1620
 -----
 1944
```

Provide the children with examples of questions that require them to add, subtract and multiply in multi-step problems.

e.g.
Below is a square tile. A rectangular tile is 3 cm longer and 2 cm narrower than the square tile. What is the difference in area between the two tiles?



Sample SATs questions

Ally and Jack buy some stickers.

Pack of 12 stickers
£10.49

12 stickers
99p each

Ally buys a pack of 12 stickers for £10.49

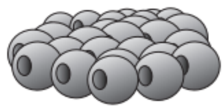
Jack buys 12 single stickers for 99p each.

How much more does Jack pay than Ally?

Layla makes jewellery to sell at a school fair.

Each bracelet has **53** beads.

She makes **68** bracelets.



Each necklace has **105** beads.

She makes **34** necklaces.

How many beads does Layla use **altogether**?

Digging Deeper

SETTING THE SCENE

Here are some digit cards. Insert these to make the number sentence true.

6 5 7 9 3

☐ 4 x ☐ 5 ☐ 3 < 200,000

EXPLORE

True of False?

- If you use the 3-digit card in the thousands column, the outcome is always true regardless of where the rest of the digit cards go
- There is a total of 5 possible true number sentences
- You can only use the 9 digit card in the tens column of the 4-digit number

Explain your reasoning.

Children are to explore which digit has the greatest impact on outcome. Why is this the case?

TAKING IT FURTHER

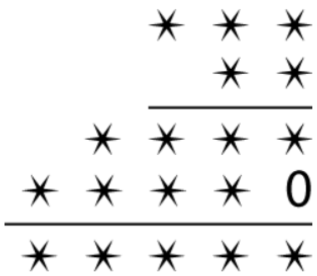
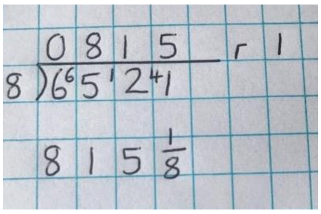
Here are some different digit cards. Create your own true or false statements.

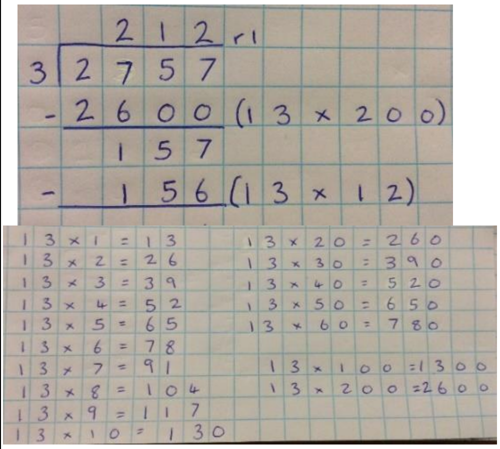
7 4 2 9 8

300,000 > ☐ 4 x 5 ☐ ☐ 3

Guide children into looking for the most influential digit card.

70 x 5000 is 350,000. Why is this going to help?

	NRICH – long multiplication A 3 digit number is multiplied by a 2 digit number and the calculation is written out as shown below. Each star like this * stands for one digit. Apart from the zero shown the only digits which occur are 2, 3, 5 and 7. Can you use this information to complete the whole multiplication? 
Written division	Recap Y5 learning if needed. Ensure that children can confidently complete written division calculations using the formal methods of short division.  Children continue to develop their use of short division and how to express remainders as whole numbers, fractions and rounded numbers. Points to consider before moving to long division. Use your school calculation policy to determine your next step for division by a 2-digit number, will you look at chunking or move straight to formal long division? Are all children ready to move onto division by a 2-digit number or do they need further time to consolidate other areas of multiplication and division? **Chunking** Supported by their secure understanding of the mental division learning done previously, introduce children to long division by chunking. Children should be taught how to set this out clearly, including noting down multiples of the number to support this process. Start with the same divisor and change just the dividend until the children become confident with the method. They should be encouraged to take away the largest 'chunk' they can each time to limit the number of steps and therefore likely errors. Children should aim to get to the answer in a maximum of 2 steps, with a remainder if needed.

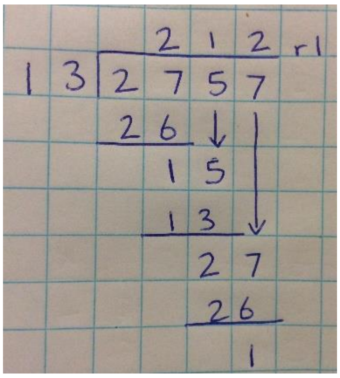


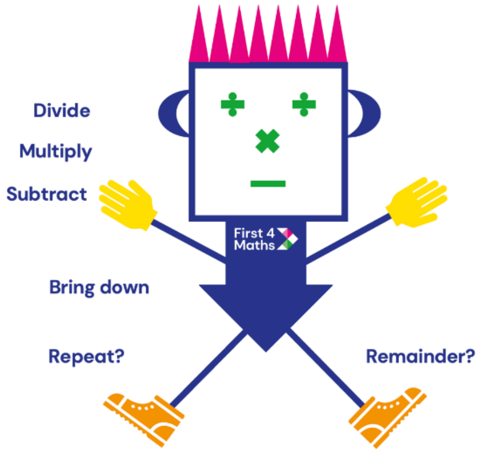
Formal methods of Long Division

When the children have a clear understanding of the place value within their division calculations, they can move onto a formal method for long division. This reduces the amount of related facts that they need to use, and therefore will improve their efficiency.

It may help to show the children how to complete a long division calculation with a 1-digit divisor alongside a short division with the same calculation. This will enable the children to see where the steps in the long division come from.

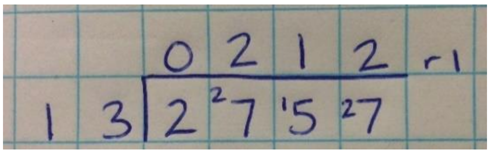
Once they are confident with this move into long division with a 2-digit divisor, again keeping with the same divisor until they are confident with the method.





Using Short Division to divide by two-digit numbers

When children are fully secure with long division for dividing by a two-digit number, they may progress to a short division method. Be aware that there are multiple parts to each step and therefore children may make errors if they rush or if their understanding is not yet secure enough.



Application to problems

Using their preferred method of division, provide children with a range of problems to solve, including past SATs questions.

Amina posts three large letters.

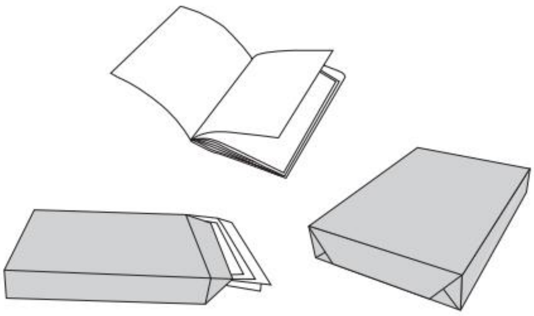
The postage costs the same for each letter.

She pays with a £20 note.

Her change is £14.96

What is the cost of posting **one** letter?

Adam is making booklets.



Each booklet must have **34** sheets of paper.

He has **2** packets of paper.

There are **500** sheets of paper in each packet.

How many complete booklets can Adam make from **2** packets of paper?

	Using the rules below, find the 2 missing numbers to make this number sentence correct. ☐ x ☐ = 1575 • Your numbers must be 2 digits and must not be a multiple of 10. • Which would be quicker: using your division knowledge or multiplication knowledge? Explain why. • Can you find another solution?
BODMAS	Ask the children to look at the question below and discuss what they think the answer is. 25 + 7 x 9 = Tell the children that the answer is 88. Ask them to try to work out how you got the answer 88. Then ask them to explore the problem again but this time with brackets. (25 + 7) x 9 and 25 + (7 x 9) Do they give the same answer? Why not? Introduce the children to the concept BODMAS. B- brackets O – order (squares and cubes) D – division M – multiplication A – addition S – subtraction Explain that this tells us the order that we complete calculations in. Brackets come first before square and cube numbers. Multiplication and division come before addition and subtraction. Can the children use this knowledge to explain why the answer was 88 in the first problem and why the answers were different when brackets were added?

Which is correct?

6 x 5 + 3 = 6 x 8 = 48

6 x 5 + 3 = 30 + 3 =33

Provide the children with fluency questions to practise the skill of calculating in the correct order.

Encourage the children to complete 'Spot the mistake' answers. You could use mistakes that the children made from the previous lesson to explore here.

4 7 8 9 5 3

Using the digits above, how many different answers can you make using BODMAS?

Ensure children can apply their understanding of square and cube numbers to tackle orders within their BODMAS work

32 $9^2 - 36 \div 9 =$

1 mark

Mastery

Compare 31 + 9 × 7 and (31 + 9) × 7
What's the same? What's different?

Choose operations to go in the empty boxes to make these number sentences true.

6 ☐ 3 ☐ 7 = 16
6 ☐ 3 ☐ 7 = 27
6 ☐ 3 ☐ 7 = 9

Put brackets in these number sentences so that they are true.

12 – 2 × 5 = 50
12 – 8 – 5 = 9
10 × 8 – 3 × 5 = 250

Greater Depth

Write different number sentences using the digits 2, 3, 5 and 8 before the equals sign, using:

- one operation
- two operations but no brackets
- two operations and brackets.

Can you write a number sentence using the digits 2, 3, 5 and 8 before the equals sign, which has the same answer as another number sentence using the digits 2, 3, 5 and 8 but which is a different sentence?

A shop sells boxes of chocolates costing £2·60. The shop also sells packets of sweets. One packet costs £1·39. Ramesh has a £10 note and he wants to buy one box of chocolates.

Sara says that Ramesh can work out how many packets of sweets he can buy using the number sentence 10 – 2·60 ÷ 1·39.

Do you agree or disagree with Sara?

If you disagree, what number sentence do you think Ramesh should use?

Explain your reasoning.

Digging Deeper

SETTING THE SCENE

Calculate the answers to these number sentences. How do the brackets change the answers?

9 x 8 + 25 ÷ (5 – 4) =	42 + (8 x 10) – 8 ÷ 2 =
9 x 8 + 25 ÷ 5 – 4 =	(42 + 8) x (10 – 8) ÷ 2 =
((9 x 8 + 25) ÷ 5 – 4) =	42 + (8 x 10 – 8) ÷ 2 =

Give children time to play with the numbers to find a range of different answers.

When will inserting brackets around part of a calculation have an impact on the answer?

TAKING IT FURTHER

Manipulate these number sentences below by adding brackets to make the more than less than statements correct. Can you create your own using all four operations?

42 ÷ 8 + 40 x 12 – 4 < 42 ÷ 8 + 40 x 12 – 4 < 42 ÷ 8 + 40 x 12 – 4

How can you use brackets to make a greater outcome?

How are you using brackets around addition and subtraction differently to multiplication and division?

	EXPLORE Using your knowledge of the order of operations, you can insert brackets into these number sentences to give a different answer. How many different answers can you come up with? Did you use a strategy? Can you be systematic? 9 x 8 + 25 ÷ 5 – 4 = 42 + 8 x 10 – 8 ÷ 2 =
Solve multi-step problems using all four operations	**Mastery** A box of labels costs £24. There are 100 sheets in the box. There are 10 labels on each sheet. Calculate the cost of one label, in pence. Miriam and Alan each buy 12 tins of tomatoes. Miriam buys 3 packs each containing 4 tins. A pack of 4 costs £1·40. Alan buys 2 packs each containing 6 cans. A pack of 6 costs £1·90. Who gets the most change from a £5 note? **Mastery with Greater Depth** A box of labels costs £63. There are 140 sheets in the box. There are 15 labels on each sheet. Sara, Ramesh and Trevor want to calculate the cost of one label, in pence. Ramesh uses the number sentence (6300 ÷ 140) × 15. Sara uses the number sentence 63 ÷ 1·4 ÷ 15. Trevor uses the number sentence (15 × 140) ÷ 6300. Who is using the right number sentence? Explain your choice. Miriam buys 19 tins of soup. All the tins cost the same price. She goes to the shop with just one note, and comes home with the tins and the change in coins. On the way home she drops the change. She looks carefully and she thinks she picks it all up. When she gets home she gives £2·23 change to her mother. Do you think that Miriam picked up all the change that she dropped? Explain your reasoning.