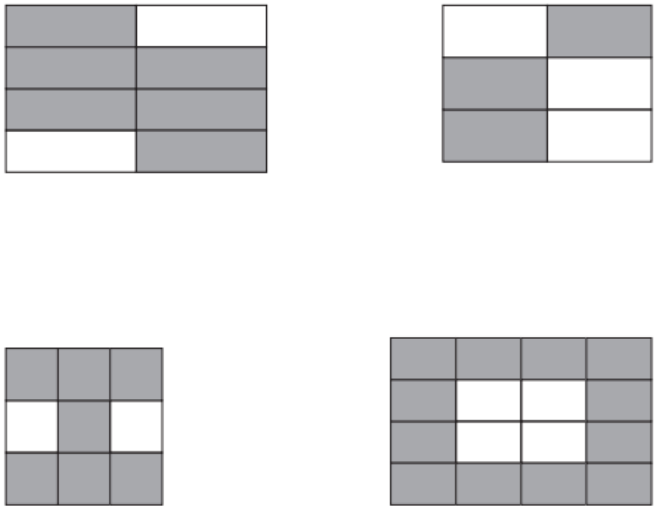
Planning Overview
Year 6 Fractions

Use common factors to simplify fractions; use common multiples to express fractions in the same denomination
Compare and order fractions, including fractions > 1
Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 4 1 × 2 1 = 8 1]
Divide proper fractions by whole numbers [for example, 3 1 ÷ 2 = 6 1]
Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 8 ÷ 3]

6F–1 Recognise when fractions can be simplified, and use common factors to simplify fractions.
6F–2 Express fractions in a common denomination and use this to compare fractions that are similar in value.
6F–3 Compare fractions with different denominators, including fractions greater than 1, using reasoning, and choose between reasoning and common denomination as a comparison strategy

	Teaching and Learning
Equivalent fractions	In Year 5, children will have found equivalent fractions where the denominators are multiples of the same number. The children will need to build on this knowledge to find equivalent fractions with different denominators. Ask children to remind you what they know about equivalent fractions. What is the rule that they learnt? How many equivalent fractions can you find to the fraction $\frac{4}{5}$? Ensure children are confident to apply their learning to be able to solve SATs style questions before moving on. e.g. $\frac{1}{3} = \frac{\square}{6} = \frac{3}{\square} = \frac{\square}{12}$

	Tick two shapes that have $\frac{3}{4}$ shaded.  Encourage the children to apply their knowledge of equivalent fractions to convert two fractions so that they have the same denominator. e.g. $\frac{2}{3}$ and $\frac{3}{4}$ What are you using your knowledge of here? Ensure the children know that they are applying their knowledge of common multiples.
Simplifying fractions	Show the children two fractions that are equivalent. $\frac{2}{5} = \frac{4}{10}$ What has happened here? The numerator and denominator of the first fraction have been multiplied by 2. Ask the children what they would have done if they started with $\frac{4}{10}$. They would have divided the numerator and denominator by 2 because 2 is a common factor. Explain that this is called simplifying. Children need to know: A fraction can be simplified when the numerator and denominator have a common factor other than 1. Explore the example $\frac{4}{12}$. Show the children that you can simplify in one step or in two.

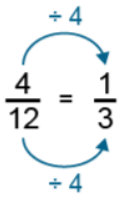


Figure 21: simplifying $\frac{4}{12}$ by dividing the numerator and denominator by the highest common factor

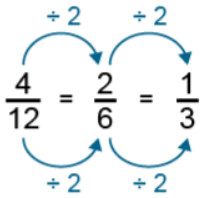


Figure 22: simplifying $\frac{4}{12}$ in 2 steps

Taken from – mathematics guidance: Key Stage 1 and 2 – Non-statutory guidance for the National Curriculum in England.

When you simplify using the highest common factor, it is more efficient as you only have to complete one step.

Provide the children with more examples of simplifying where the numerator is a factor of the denominator. Encourage them to notice that the numerator is the highest common factor.

e.g. $\frac{3}{9}$ $\frac{7}{28}$ $\frac{12}{36}$

6 is not a factor of 15 so how could you simplify this fraction?

$$\frac{6}{15}$$

Encourage the children to use the term 'highest common factor'. The highest common factor of 3 and 15 is 3 so you can divide the numerator and denominator by 3 to simplify the fraction to $\frac{1}{5}$.
Give the children time to practise simplifying fractions where the numerator is not a factor of the denominator.

e.g. $\frac{8}{18}$ $\frac{12}{20}$ $\frac{9}{12}$

Always/sometimes/never
To simplify a fraction you just half the numerator and denominator until you can't halve them anymore. Is this always, sometimes or never true?

Jasper thinks that these fractions are equivalent because all of the numbers have 5 as a common factor. Do you agree? Explain.

$$\frac{15}{20} = \frac{25}{50}$$

He thinks these are equivalent because all of the numbers have a common factor of 3. Do you agree? Explain.

$$\frac{14}{18} = \frac{21}{27}$$

Allow the children to apply their knowledge to simplifying improper fractions and mixed numbers.
e.g.

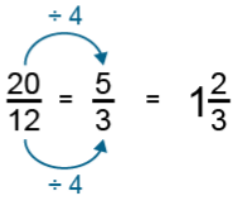


Figure 23: simplifying $\frac{20}{12}$ to $\frac{5}{3}$, then converting to a mixed number

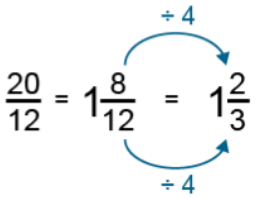


Figure 24: converting $\frac{20}{12}$ to $\frac{5}{3}$, then simplifying

Taken from – mathematics guidance: Key Stage 1 and 2 – Non-statutory guidance for the National Curriculum in England.

Children should learn that when the numerator and denominator of a fraction have no common factors (other than 1) then the fraction is in its simplest form.

Ask the children how can you ensure that you have converted a fraction into its simplest form?
"To convert a fraction to its simplest form, divide both the numerator and the denominator by their highest common factor."

Provide children with a set of fractions and ask them to sort them whether they are in their simplest form or not, including mixed and improper fractions.

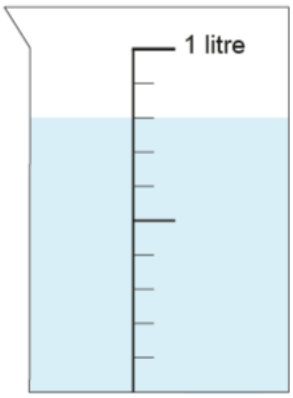
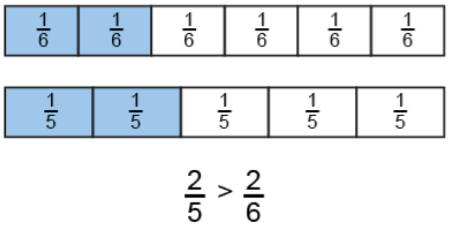
In its simplest form	Not in its simplest form

Ask them to prove why they are not in their simplest form.

Sample SATs question

Write the two missing values to make these equivalent fractions correct.

$$\frac{\square}{30} = \frac{10}{12} = \frac{30}{\square}$$

	How much water is in this beaker? Write your answer as a fraction of a litre in its simplest form.  Taken from – mathematics guidance: Key Stage 1 and 2 – Non-statutory guidance for the National Curriculum in England.
Compare fractions including fractions >1.	In Year 5, children will have learnt how to convert between improper fractions and mixed numbers. Ensure the children have retained this knowledge before moving on. First explore which fraction is larger when the numerators are the same. $\frac{2}{5}$ or $\frac{2}{6}$ Which fraction is larger? How do you know? You could show this with a bar model to support.  $\frac{2}{5} > \frac{2}{6}$ **Figure 27: bar models to compare $\frac{2}{5}$ and $\frac{2}{6}$** Taken from – mathematics guidance: Key Stage 1 and 2 – Non-statutory guidance for the National Curriculum in England. When the numerators are the same, we can order the fractions without doing any calculating. Remind children that the larger the denominator, the smaller the fraction. Therefore, $\frac{3}{7}$ is less than $\frac{3}{5}$. Jess writes this. Explain how you know she is correct. $\frac{3}{4} > \frac{3}{7}$

Mastery

Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer.

Explain your reasoning.

Move on to questions where the numerators are different. Children should be confident converting fractions into the same denominator where one is a multiple of the others.
e.g. $\frac{1}{5}$ and $\frac{4}{15}$.

The children will now need to find common multiples to find common denominators in order to be able to compare.
e.g. $\frac{1}{3}$ and $\frac{3}{8}$.
8 is not a multiple of 3 but 24 is a common multiple of 3 and 8. The common denominator will be 24. The children will then need to convert both fractions to have a denominator of 24.

Position the symbols <, > or = to make these statement correct.

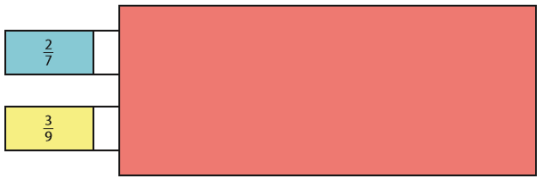
$$\frac{3}{4} \square \frac{2}{3}$$

$$1\frac{4}{5} \square 1\frac{7}{8}$$

$$\frac{11}{9} \square \frac{6}{5}$$

Now provide the children with worded problems where they have to compare amounts.
e.g.
Jasmine and Imran are raising money for charity. Jasmine has raised $\frac{7}{10}$ of the total amount. Imran has raised $\frac{3}{5}$ of the amount. Who is the closest to their target?

Would you rather?
On a really hot day, would you rather drink $\frac{3}{4}$ of a bottle of water or $\frac{5}{7}$ of a bottle of water?

	Sample SATs style question. Tick the fractions **less than** $\frac{5}{8}$ $\frac{1}{2}$ ☐ $\frac{2}{8}$ ☐ $\frac{3}{4}$ ☐ $\frac{7}{16}$ ☐ $\frac{24}{32}$ ☐ **Mastery with Greater Depth** Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer  Explain your reasoning.
Order fractions including fractions >1.	The children can now apply their knowledge of equivalent fractions and simplifying to order fractions with different denominators where there are more than 2 fractions. Ask children to roll a 1-12 dice four times to create a set of three fractions. They must then order them from the smallest to largest. Simplify these fractions first. What do you notice? Why will it now be easy to order them from largest to smallest? $\frac{3}{18}$ $\frac{5}{20}$ $\frac{4}{8}$ $\frac{2}{18}$ $\frac{4}{12}$ $\frac{6}{60}$ Sample SATs style question Here are four fraction cards. $\frac{3}{4}$ $\frac{5}{8}$ $\frac{6}{12}$ $\frac{7}{16}$ Use any **three** of the cards to make this correct. ☐ < ☐ < ☐

	Now give examples where there are several improper fractions and several mixed numbers to order. e.g. Order these in ascending order. $\frac{5}{4}$ $\frac{7}{2}$ $\frac{8}{5}$ $\frac{19}{10}$ Order these in descending order. $1\frac{1}{4}$ $1\frac{2}{3}$ $1\frac{5}{6}$ $2\frac{1}{3}$ Ask the children to complete questions where there is a mix of proper fractions, improper fractions and mixed numbers where the denominators differ. $1\frac{2}{3}$ $\frac{7}{12}$ $\frac{7}{8}$ $\frac{8}{3}$ $\frac{23}{24}$ $1\frac{9}{12}$ Can they order any without converting to a common denominator? Explain. Find two fractions that make this correct. The denominators have to be different. Find three different examples. $\square > \frac{4}{7} > \square$ Add fractions to the boxes below so that they are ordered from the smallest to the largest. $\frac{3}{8}$ $\square$ $\frac{1}{2}$ $\frac{7}{10}$ $\square$ $\frac{4}{5}$
Add and subtract fractions	In Year 5, the children will have practised adding and subtracting fractions whose denominators are all multiples of the same number, including mixed numbers. Show the children the question below. Why is this difficult? Ask them to discuss how they can use their previous learning to help them to solve the problem. $\frac{1}{2} + \frac{1}{3} =$ The children will need to convert them both into fractions with a common denominator and then they can add the fractions.

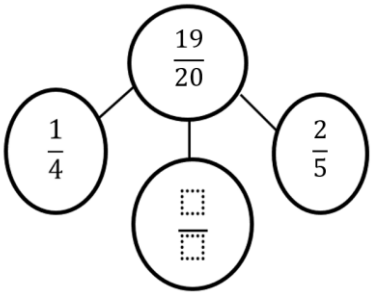
Why is the answer not $\frac{2}{5}$?

How would this work with subtraction?

$$\frac{2}{7} - \frac{1}{9} =$$

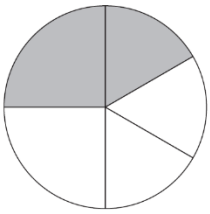
Allow the children to practise adding and subtracting proper fractions together.
As an extra challenge, ask the children to ensure all of their answers are in their simplest form.

Complete the part-whole model. Can you write the answer in its simplest form?



Sample SATs question.

In this circle, $\frac{1}{4}$ and $\frac{1}{6}$ are shaded.



What fraction of the whole circle is **not** shaded?

Mastery

Sam added two fractions together and got $\frac{7}{8}$ as the answer.
Write down two fractions that Sam could have added.

Tom wrote down two fractions. He subtracted the smaller fraction from the larger and got $\frac{1}{5}$ as the answer.
Write down two fractions that Tom could have subtracted.

Tom and Sam shared equally one third of a chocolate bar.
What fraction of the chocolate bar did each child get?

Can you find three unit fractions that add together to make 1?

Now progress to adding and subtracting mixed numbers.

Ask the children to add and subtract proper fractions to mixed numbers. What do we need to do before we can add or subtract?

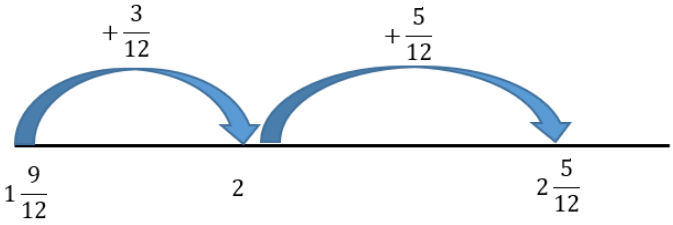
$1\frac{3}{4} + \frac{2}{3} =$

The children will need to convert the fractions so that they share a common denominator.

$\frac{3}{4} = \frac{9}{12}$ $\frac{2}{3} = \frac{8}{12}$

$1\frac{9}{12} + \frac{8}{12} =$

Explore the use of a number line to add the fractions.



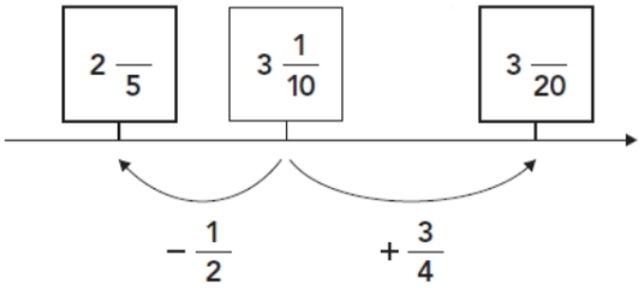
Complete the same process with subtracting a proper fraction from a mixed number.

Allow the children time to practise this skill with a range of fluency questions.

Apply knowledge to solving problems

Alife and Ellie bought some pizzas. Ellie ate $1\frac{3}{5}$ of a pizza more than Aflie. Aflie at $\frac{5}{6}$ of a pizza. How much did they eat altogether?

Sample SATs question
Fill in the missing numbers.



The numbers in this sequence increase by the same amount each time.

Write the missing numbers.

	1	$1\frac{5}{8}$	$2\frac{1}{4}$	

Now explore how to add and subtract mixed numbers using a number line.

e.g. $4\frac{2}{3} - 1\frac{6}{7} =$

As there is large room for error, first ask the children to make an estimate of what the answer will be. Use the strategies from addition and subtraction and multiplication to discuss how you can round mixed numbers to help to make estimations.

Sample SATs question

Layla wants to estimate the answer to this calculation.

$$3\frac{9}{10} - 2\frac{1}{8} + 1\frac{4}{5}$$

Tick the calculation below that is the best estimate.

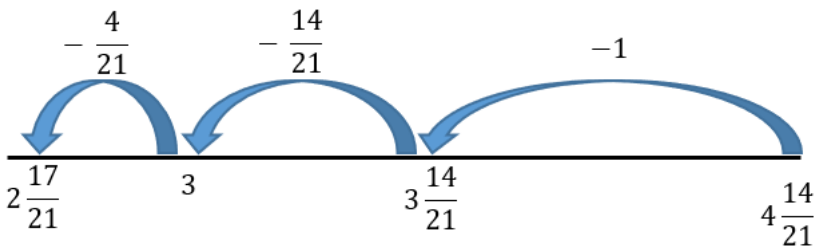
Tick **one**.

- 3 – 2 + 2 ☐
- 4 – 2 + 1 ☐
- 4 – 2 + 2 ☐
- 3 – 2 + 1 ☐

When finding the exact answer, the children need to convert the fractions so that they have a common denominator.

$$4\frac{2}{3} = 4\frac{14}{21} \qquad 1\frac{6}{7} = 1\frac{18}{21}$$

They can then use a number line to compelte the subtraction.



Ask the children to explore different strategies for adding and subtracting mixed numbers. Which method is the most efficient and why?

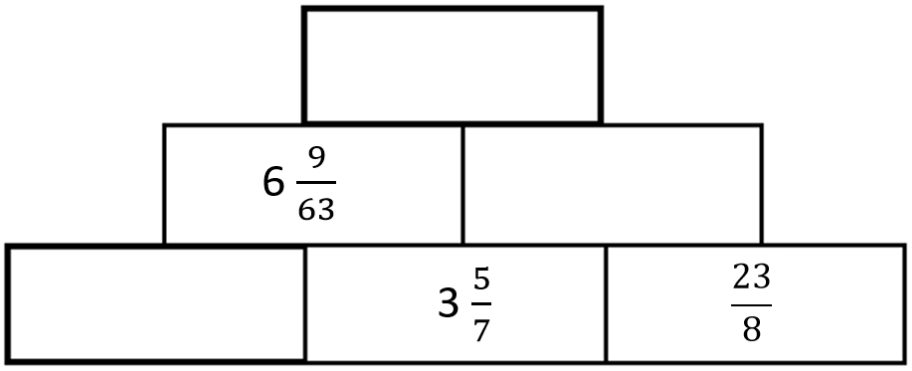
Mastery

On Monday I ran $1\frac{2}{3}$ km and on Tuesday I ran $2\frac{2}{5}$ km.
How far did I run altogether on these two days?

On Wednesday I ran $1\frac{2}{3}$ km and my sister ran $2\frac{2}{5}$ km.
How much further did my sister run than I did?

Apply knowledge to solving more problems.
Grace, Ellie and Alfie bought some pizzas to share. Grace ate $1\frac{7}{12}$ pizzas more than Alfie. Ellie ate $1\frac{2}{3}$ pizzas. Alfie ate $\frac{3}{4}$ more than Ellie.
How many pizzas do they eat altogether?

The number in each box is the sum of the two numbers below it. Write in the missing numbers.



Mastery with Greater Depth

Altogether on Monday and Tuesday I ran $3\frac{1}{2}$ km. On neither day did I run a whole number of km.
Suggest how far I ran on Monday and how far on Tuesday.

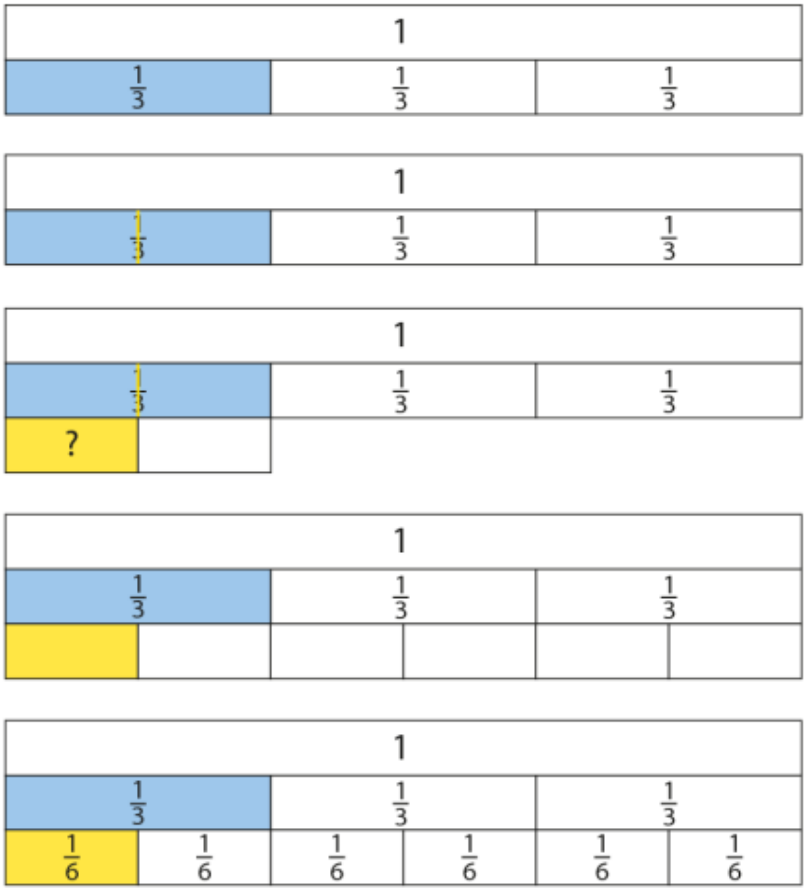
On Wednesday I ran some km and my sister ran $1\frac{1}{6}$ km further than I did.
Altogether we ran $4\frac{1}{2}$ km.

How far did I run on Wednesday?

Multiplying pairs of proper fractions	Ensure that children have recalled how to multiply proper fractions and mixed numbers by whole numbers from Year 5. **Paper folding** To find $\frac{3}{4}$ x $\frac{1}{2}$ we are finding $\frac{3}{4}$ of $\frac{1}{2}$ Ask children to fold a piece of A4 paper in half lengthways and shade $\frac{1}{2}$ of the paper: Now fold the same paper into quarters cross ways by folding in half and half again to make four equal parts and shade $\frac{3}{4}$ in a different colour: The paper now shows $\frac{3}{4}$ of $\frac{1}{2}$ in the parts that are shaded in both colours. Ask children how many parts are shaded in both colours? 3 How many parts are there in total? 8 3 out of 8 parts are shaded so $\frac{3}{8}$ is the answer to $\frac{3}{4}$ of $\frac{1}{2}$ Repeat with other examples.

Now get children to explore this concept with a bar model. *Note that of means the same as x.*

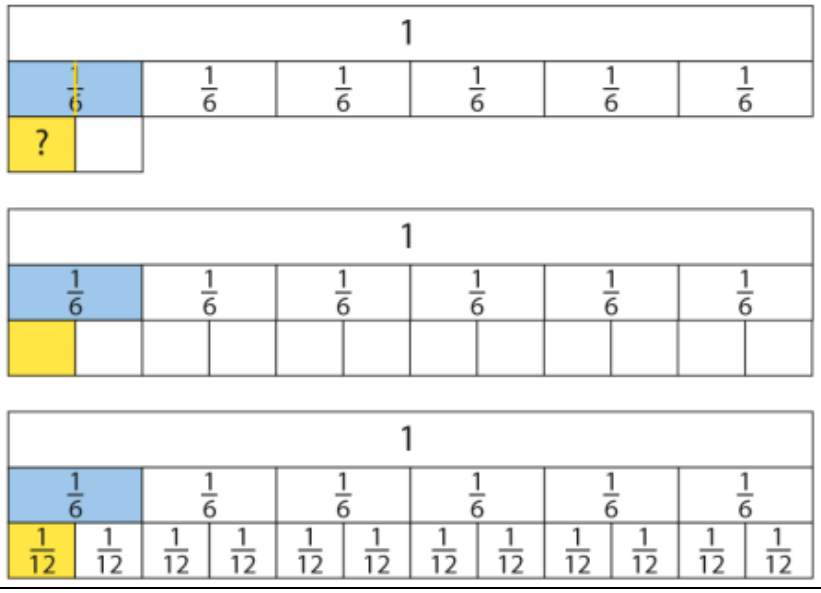
e.g. Find $\frac{1}{2}$ of $\frac{1}{3}$ (this could be written as $\frac{1}{2} \times \frac{1}{3}$)



Taken from NCETM PD materials

Again complete with other examples.

What question has been represented here?



	Taken from NCETM PD materials Provide children with a list of calculations that they have represented in this way. What do they notice? e.g. $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ $\quad$ $\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$ $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ $\quad$ $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$ $\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$ $\quad$ $\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$ $\frac{1}{3}\times\frac{1}{5}=\frac{1}{15}$ $\quad$ $\frac{1}{5}\times\frac{1}{3}=\frac{1}{15}$ What do children notice? Encourage them to see that the numerators have been multiplied and so have the denominators. Repeat the activity for other pairs of fractions where the numerators are note one, including where a simplification of the final fraction is needed. $\frac{3}{4}$ x $\frac{2}{3}$ = $\frac{6}{12}$ = $\frac{1}{2}$ James says that the answer to the question below is one quarter. Is he correct? Explain your thinking. $\frac{1}{3}$ x $\frac{3}{4}$ = **Always/sometimes/never** Steven says that when you multiply two proper fractions, the answer gets smaller. Is this always, sometimes or never true?

'Fill in the missing symbols (<, > or =).'

$\frac{1}{2} \times 6 \bigcirc \frac{1}{2}$

$\frac{1}{2} \times 1 \bigcirc \frac{1}{2}$

$\frac{1}{2} \times \frac{1}{6} \bigcirc \frac{1}{2}$

$\frac{1}{2} \times \frac{1}{6} \bigcirc \frac{1}{6}$

'How many solutions can you find to make the statement true?'

$$\frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{3}{20}$$

Taken from NCETM PD materials

Mastery with Greater Depth

True or false?

- The sum of two fractions is always greater than their product.
- If I divide a fraction by a whole number, the quotient is always smaller than the dividend.

Explain your reasoning.

Complete the first bullet point from the question.

Digging Deeper

SETTING THE SCENE

Here are two fractions multiplied by each other. The answer has been given in its simplest form.
Can you find two fractions that can be multiplied to make this statement correct?

$$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{7}{16}$$

EXPLORE

Find as many possibilities as you can for solving the above using four different digits in each calculation. Which digit cards cannot be used? Why can these not be used?

Guide children into looking at multiples of 7 and 16.

Which equivalent fractions could be used instead of 7/8? How will this help you?

Suggested answers

$\frac{7}{8} \times \frac{2}{4} = \frac{21}{48} = \frac{7}{16}$

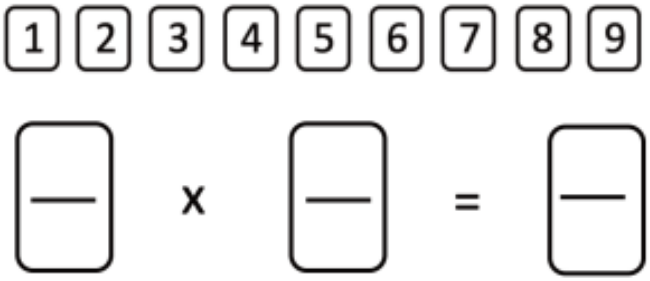
$\frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$ $\quad\quad$ $\frac{7}{8} \times \frac{2}{4} = \frac{7}{16}$

1 2 3 4 5 6 7 8 9

TAKING IT FURTHER

Do you agree or disagree with this statement?

There are no proper fractions that can be multiplied together when finding all possibilities, that will use all of the digit cards.



Which proper fraction will this start with?

Encourage children to work systematically starting with the answer and working backwards.

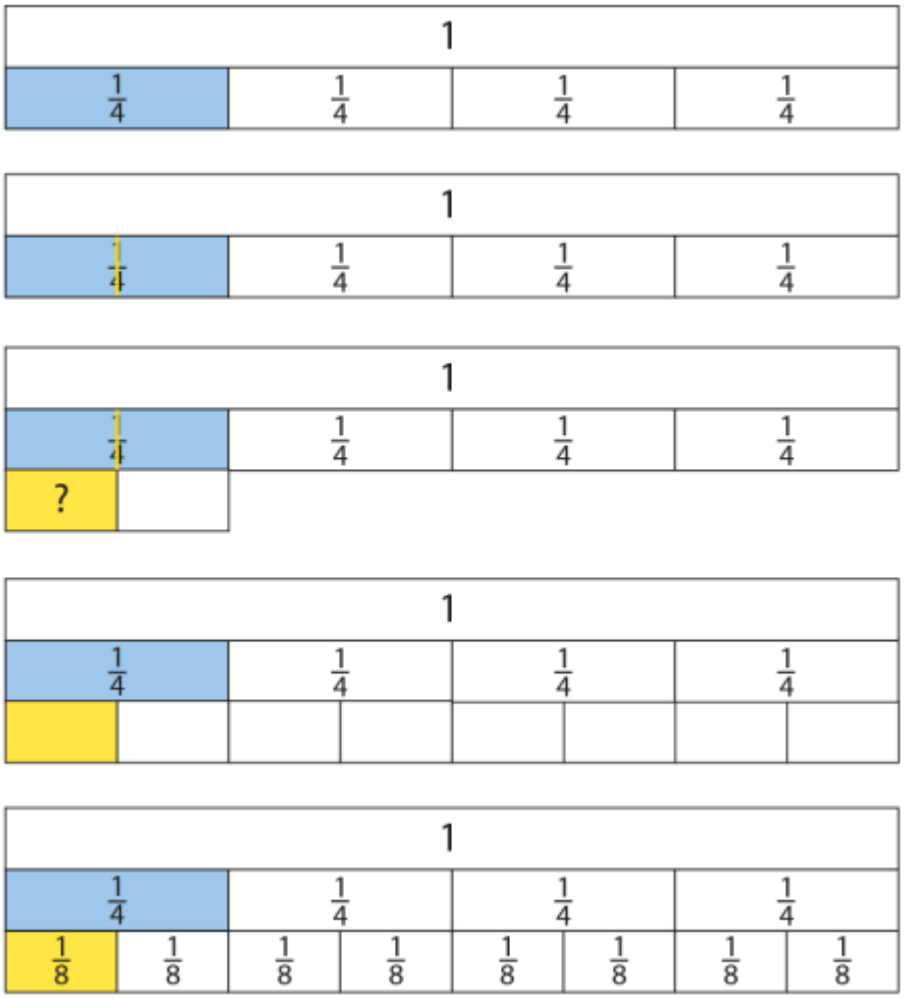
Suggested starting point

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{1}{2}$ $\quad$ $\frac{\square}{\square} \times \frac{\square}{\square} = \frac{2}{4}$ $\quad$ $\frac{\square}{\square} \times \frac{\square}{\square} = \frac{3}{6}$

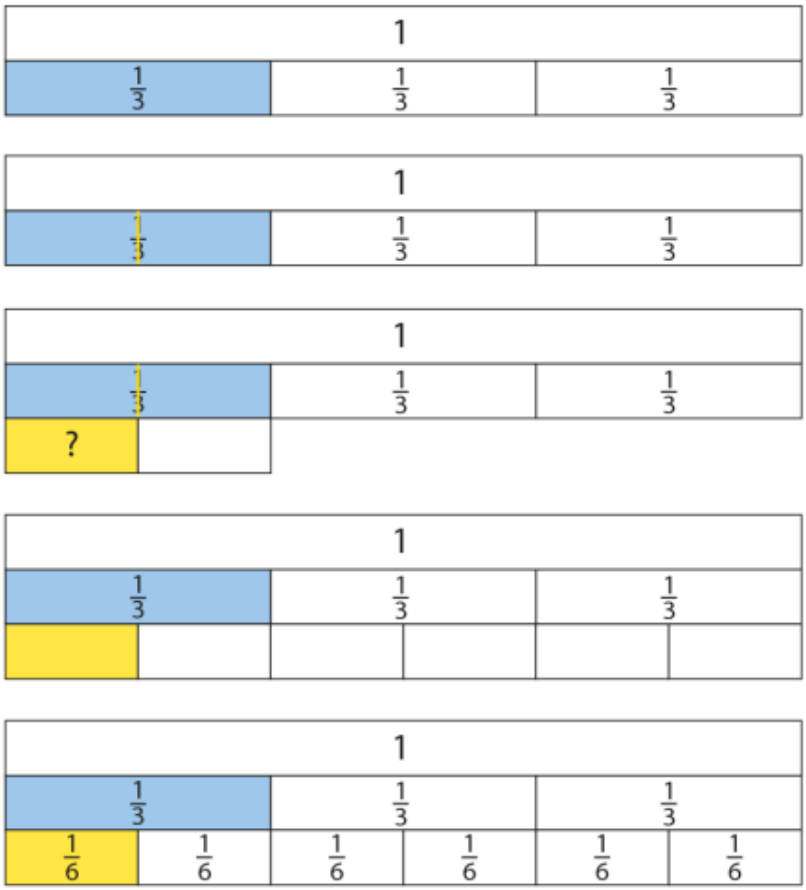
As the children work through the possibilities, they will see that the digit 5 and 9 cannot be used as part of the answer.

OUTCOME

Children will recognise the correlation between factors and multiples when multiplying fractions by fractions.

Divide proper fractions by whole numbers	Use the following representation to recap what the children have learnt about multiplying fractions. There are two ways that they know of to represent one half of $\frac{1}{4}$. $\frac{1}{4} \times \frac{1}{2}$ or $\frac{1}{2} \times \frac{1}{4}$  *Taken from NCETM PD materials* Ask the children if they can think of another way to halve numbers in maths. Children should be able to say that you can divide by two. Show them the equation $\frac{1}{4} \div 2 =$. Discuss with the children how it relates to the bar models above. $\frac{1}{4} \div 2 = \frac{1}{8}$ Look at the two methods side by side. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ $\frac{1}{4} \div 2 = \frac{1}{8}$ What is the same? What is different?

Complete this process with this image. Can the children apply their learning?



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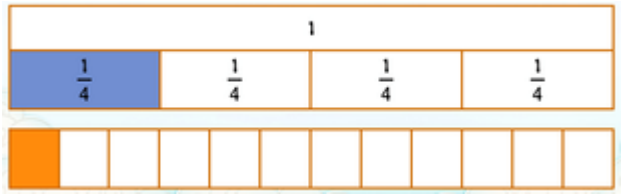
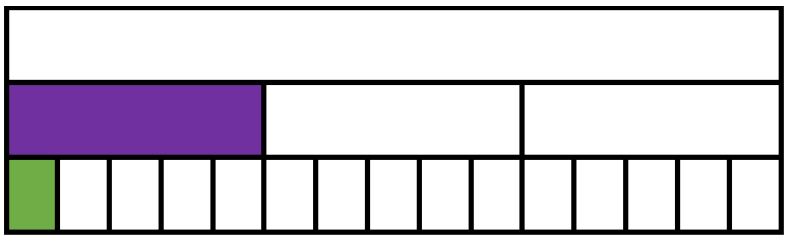


This bar model represents $\frac{1}{4} \div 3 = \frac{1}{12}$.
Ask children to explain how it shows this.
(e.g. $\frac{1}{4}$ of the whole is shaded. The $\frac{1}{4}$ is divided into 3 parts and the other $\frac{3}{4}$ are also divided into 3 parts each. There are 12 smaller parts, and one is shaded so $\frac{1}{4} \div 3 = \frac{1}{12}$)

What does this bar model show?



Draw a diagram to represent this division:

$\frac{1}{4} \div 3 =$

Ask the children what division question can be used to represent the following multiplication.

$$\frac{1}{3} \times \frac{1}{4} =$$

Provide the children with fluency questions to divide proper fractions by whole numbers.

'Fill in the missing numbers.'

$\frac{1}{8} \div 3 = \frac{1}{8} \times \square$ $\quad$ $\frac{1}{4} \div 4 = \square \times \square$

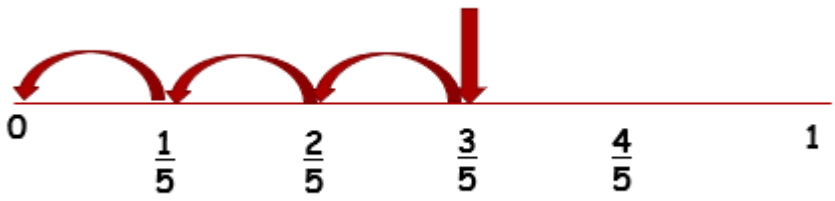
$\frac{5}{9} \div 2 = \frac{5}{9} \times \square$ $\quad$ $\frac{2}{3} \div 7 = \square \times \square$

$\frac{1}{7} \times \frac{1}{4} = \frac{1}{7} \div \square$ $\quad$ $\frac{3}{5} \times \frac{1}{2} = \frac{3}{5} \div \square$

Taken from NCETM PD materials

Now move on to examples where the numerator is a multiple of the divisor. Show the children that another way to use pictures to support division of fractions is a number line – this helps to make the link with repeated subtraction and reinforces the concept of a fraction as a number with a numerical value.

Draw a number line from 0 to 1.
Divide the number line into the number of parts for your fraction e.g. five parts for fifths.
Mark your fraction on the number line.
Jump back (or forwards) in equal steps to (or from) zero.
Use your number line to work out the value of each step.



Ask children how this number line shows that $\frac{3}{5} \div 3 = \frac{1}{5}$. Each of the 3 repeated subtraction steps represent $\frac{1}{5}$. When would this method work and when would it not? Try to get the children to spot that the numerator is a multiple of the divisor. Together go through some examples where the numerator is a multiple of the divisor.

Always/sometimes/never
Ben says that when you divide a proper fraction by a whole number, you only divide the numerator. Is this always, sometimes or never true? Explain.
(Here you will be looking for the children to explain that when the numerator is a multiple of the divisor, this method would work. However, when it is not, the previous method is needed.)

Use the digits 1, 2, 3, 4 and 5 to make a proper fraction and a whole number to divide:
$\frac{?}{?} \div ?$

What is the biggest quotient (answer) you can make? What is the smallest?

Mastery

In each number sentence, replace the boxes with different whole numbers less than 20 so that the number sentence is true.

$\frac{1}{\square} \times \frac{3}{\square} = \frac{\square}{\square}$

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{8}{15}$

$\frac{2}{\square} \times \frac{5}{\square} < \frac{10}{\square}$

$\frac{\square}{\square} \div 3 = \frac{1}{\square}$

$\frac{\square}{\square} \div 3 > \frac{1}{4}$

<table>
<tr>
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Now ask the children to sort these calculations and explain why they have chosen to put them in each of the sections. Ensure that they solve the calculations also.

$\frac{12}{15} \div 3$ $\quad \frac{12}{15} \div 5$ $\quad \frac{6}{7} \div 3$ $\quad \frac{6}{7} \div 4$
$\frac{7}{18} \div 2$ $\quad \frac{10}{11} \div 5$ $\quad \frac{56}{65} \div 7$ $\quad \frac{54}{56} \div 7$

<table>
<tr><th>Numerator is a multiple of the divisor</th><th>Numerator is not a multiple of the divisor</th></tr>
<tr><td></td><td></td></tr>
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<table>
<tr><th>Mastery with Greater Depth</th></tr>
<tr><td>True or false?
■ The sum of two fractions is always greater than their product.
■ If I divide a fraction by a whole number, the quotient is always smaller than the dividend.

Explain your reasoning.</td></tr>
</table>

Complete the second bullet point of this question.
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<tr>
<td>Associate a fraction with division and calculate decimal fraction equivalents.</td>
<td>If you have children who are confident with the formal written method of division and secure in their understanding of applying this to decimals, you may want to cover this here. If not, this will be covered in the decimals and percentages unit.</td>
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