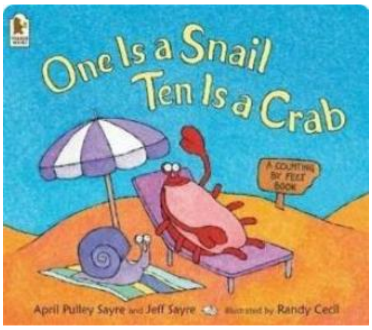
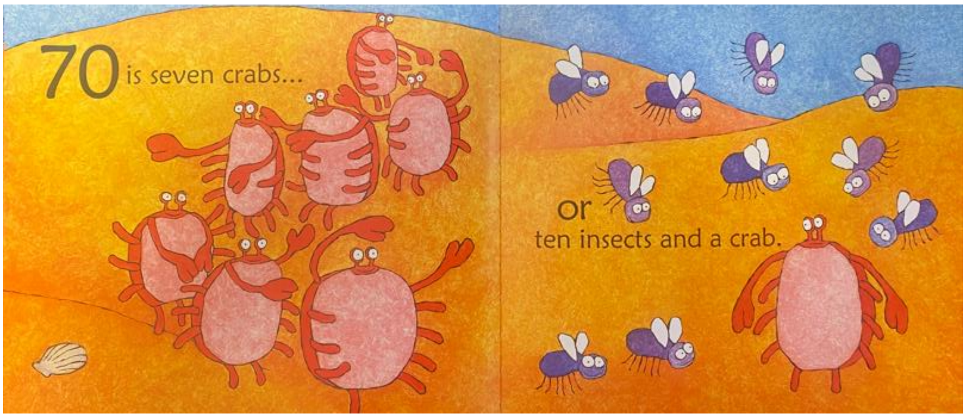
Planning Overview
Year 6 Algebra

Use simple formulae
Generate and describe linear number sequences
Express missing number problems algebraically
Find pairs of numbers that satisfy an equation with two unknowns
Enumerate possibilities of combinations of two variables.

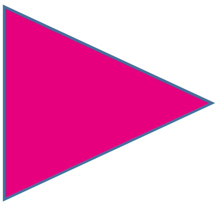
6AS/MD–4 Solve problems with 2 unknowns

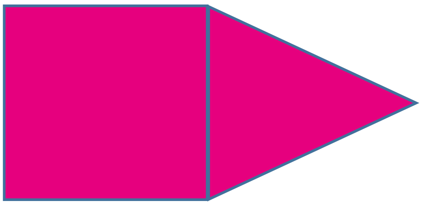
Introduction to algebra	 Read the story, 'One is a snail, Ten is a Crab' or watch a version on YouTube. Show children how we could represent each number with a letter based on the book 1 = snail = s 2 = person = p 4 = dog = d 6 = insect = i 8 = octopus = o (this has been changed from the spider as we already had s for snail) 10 = crab = c Share the rules of algebra with the children, explain that we are trying to be as efficient as possible. Use **lower case letters** n = number **Adding** a + b **Subtracting** a – b **Multiplying** – don't use x, write number first, then letter e.g. 5n **Dividing –** use fraction notation instead of ÷ e.g. $\frac{5}{n}$ means 5 ÷ n Children can be easily confused with 2n and n^2 so reinforce the difference between these two terms. 2n = n + n **or** n x 2 n^2 = n x n How would we show a person add a dog? p + d What would be the total of this? 6 What about a crab plus an insect? c + i = 16 How could we represent 3, 5, 7 and 9? e.g. 5 = d + s Is there another way we could represent 5? 5 = 5s 5 = 2p + s

<table>
<tr>
<td></td>
<td>How many ways could you represent 10?



How would you record the combinations on the page above?
I can see 50 feet, what possible combinations could I have?
Find 5 possibilities.</td>
</tr>
<tr>
<td>Use simple formulae</td>
<td>Simple formula using shape

This is shape a. This is shape b.
 

How would you describe this shape?

And this one?


What would the shapes below look like?
2a + 3b
a – b
3a – 2b</td>
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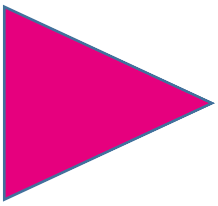
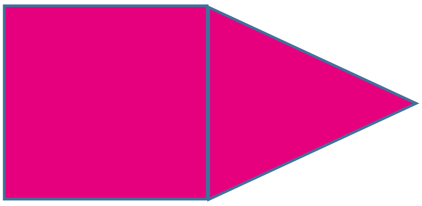
Tell the children that the shapes now have a value.

Shape a = 10

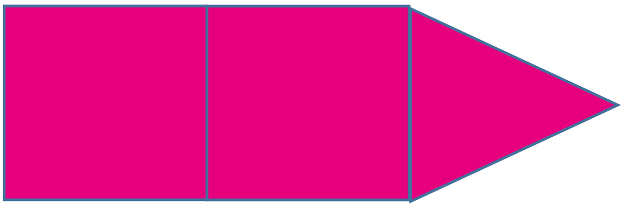
Shape b = 4



What is the value of the shape below?

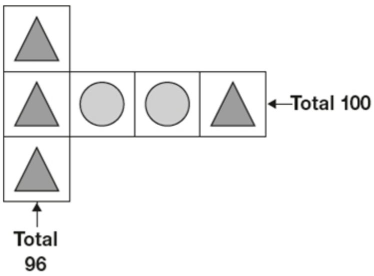


What about this one?



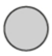
Apply understanding to SATs questions.

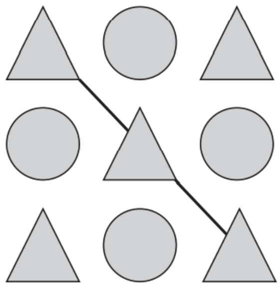
Each shape stands for a number.



Work out the **value** of each shape.

 = ______

 = ______



Each shape stands for a number.

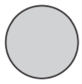
The total of the shapes on the diagonal line is 48

The total of all the shapes is 200

Calculate the value of each shape.

 =

1 mark

 =

1 mark

Simple formula using letters and numbers

Support the children in calculating simple formula where they have to change a letter into a value.
e.g. If n = 10 what are the totals of the following formulae?

n + 2

$\frac{n}{2}$ + 7

2n + 4

3n – 6

What if the value of n is 16? What would the totals be now?

Try a range of similar SATs questions.

n = 22

What is 2n + 9?

1 mark

	Extend to questions where the children need to substitute in a value to solve the problem.

Jack hires a hall for a party.

This formula is used to work out the total cost.

Total cost = £15 booking fee + £12.50 per hour

What is the total cost of hiring the hall from 6 pm until 11 pm?

£

1 mark

A shop prints designs on T-shirts.

They use this formula to work out the price for printing a design.

price = 60p × number of colours + £1.25

What is the price for printing a design that has **3** colours in it?

£

1 mark

Children should be able to reason about how to solve an algebraic equation and what they mean like in the example below.

If x = 5, y = 4 and z = 9, calculate the value of the following expression
5x + 3y – 2z

'5x means 5 multiplied by x. If x is 5 then that means 5 multiplied by 5.

I need to add this 25 to the total of 3y. 3y means 3 multiplied by y and y is 4. 3 multiplied by 4 is 12. My running total is 37.

I now need to take away 2z from this total, z is 9 so 2z will be 2 x 9 which is 18.

When I take 18 off 37, I get my final answer of 16.

Express missing number problems algebraically

Show the children a bag and tell them there are a number of balls in there. Add three more balls into the bag. How would we represent what is in the bag now? b + 3

Return to the original bag, take 2 balls out of the bag and ask the children to represent the total. b – 2

Can children apply this to the test questions below?

1 (a) There are n counters in Alfie's bag.

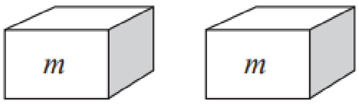
Alfie puts **3** more counters in the bag.

Write an expression for the number of counters that are in the bag now.

1 mark

(b) Megan has two boxes.

There are m counters in each box.

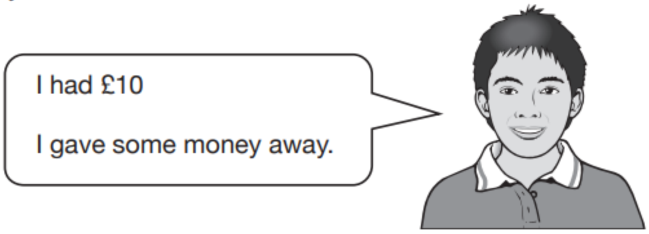


She puts all her counters together in a pile, then removes **5** of them.

Write an expression for the number of counters that are in the pile now.

1 mark

Dev says,



Which expression shows how much money Dev has left?

a is the amount of money, in pounds, that Dev gave away.

Tick **one**.

- $10 + a$ ☐
- $10 \div a$ ☐
- $a - 10$ ☐
- $10 - a$ ☐
- $a \times 10$ ☐

1 mark

k stands for a number.

Complete the number sentences below.

One has been done for you.

5 more than k is $k + 5$

2 less than k is ______

3 more than twice k is ______ 23i

6 more than half of k is ______ 23ii

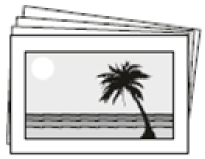
2 marks

	A theme park sells tickets online. Each ticket costs £24 There is a £3 charge for buying tickets. Which of these shows how to calculate the total cost, in pounds? Tick **one**. number of tickets × 3 + 24 ☐ number of tickets × 24 + 3 ☐ number of tickets + 3 × 24 ☐ number of tickets + 24 × 3 ☐ 1 mark **I think of a number** Can children start to represent 'I think of number' questions algebraically in preparation for the next objective? I think of a number multiply by 3 and add 4, the answer is 13. What was my starting number. if we wrote it how it is said it would be n x 3 + 4 = 13 algebraically this would be written 3n + 4 = 13 Can children create their own, "I think of a number…" questions for their partner write algebraically?
Finding unknowns in algebraic equations	Ask children to look at this problem and elicit the known information from it. **3a + 15 = 45** We know the whole is 45 and we know that we have 3 equal unknown parts and a known part of 15. Ask children to consider how we use the known information to help us to establish the unknown information. 'We need to subtract 15 from the 45 and then divide what is left into 3 parts. This will tell us what a is.' <table><tr><td colspan="4">45</td></tr><tr><td>a = 10</td><td>a=10</td><td>a=10</td><td>15</td></tr></table> Children to solve similar simple algebraic equations e.g. Test questions below. $2q + 4 = 100$ Work out the value of q.

First 4 Maths

Alfie has some photographs printed.

The cost is £2.50 for postage and
12 pence for each print.



Alfie uses this formula for the total cost (**C**) in pence.

$$\mathbf{C} = 250 + 12n$$

n stands for the number of photographs.

The total cost for Alfie is **£6.70**

How many photographs does he have printed?

See if children can apply what they know to this balancing equation.

2a + 7 = a + 11

a	a	7
a	11	

What must the value of a be?

Mastery

Which of the following statements do you agree with? Explain your decisions.

- The value 5 satisfies the symbol sentence $3 \times \square + 2 = 17$
- The value 7 satisfies the symbol sentence $3 + \square \times 2 = 10 + \square$
- The value 6 solves the equation $20 - x = 10$
- The value 5 solves the equation $20 \div x = x - 1$

Mastery with Greater Depth

Which of the following statements do you agree with? Explain your decisions.

- There is a whole number that satisfies the symbol sentence $5 \times \square - 3 = 42$
- There is a whole number that satisfies the symbol sentence $5 + \square \times 3 = 42$
- There is a whole number that solves the equation $10 - x = 4x$
- There is a whole number that solves the equation $20 \div x = x$

To enumerate possibilities of combinations of two variables

Children have calculated the number of possibilities at the end of the Ratio and Proportion unit. Can the children work systematically to find all of the possibilities and identify missing possibilities.

2 Adam chooses the colours for a new team shirt.

The shirt has **two** colours.

There are four colours to choose from: **yellow**, **blue**, **white** and **red**.

Write the **two** missing combinations.

The shirt could be:

- yellow and blue
- yellow and white
- yellow and red
- blue and white.

______________ and ______________

______________ and ______________ 1 mark

Now extend to missing values within algebraic equations.

$$3a + b = 45$$

How many unknowns do we have?
a is unknown,
b is unknown,
we know the whole is 45.

If we put this into a bar model we can see this in a less abstract way.

45			
a	a	a	b

Ask children to give a sensible assigned value to a. what will that make the value of b?

45			
a =2	a = 2	a = 2	b = 39

Ask children to explore the variety of answers that they can get for these 2 unknown variables. Can they create a table to record their answers? Can they find a way to work systematically?

a	b	Total
1	42	45
2	39	45
3	36	45

How will the children know when they have found all the combinations to solve what the 2 unknowns are?

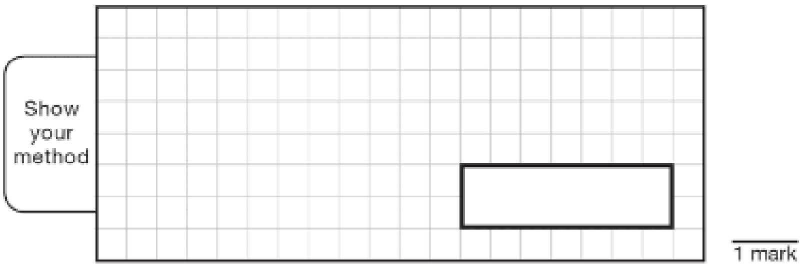
Ask children to tackle these questions from the Ready to Progress guidance using the same thinking.

1. A baker is packing 60 cakes into boxes. A small box can hold 8 cakes and a large box can hold 12 cakes.
   a. How many different ways can he pack the cakes?
   b. How can he pack the cakes with the fewest number of boxes?

5. A rectangle with side-lengths *a* and *b* has a perimeter of 30cm. *a* is a 2-digit whole number and *b* is a 1-digit whole number. What are the possible values of *a* and *b*?

Problem solve using money and measure problems with 2 unknowns

The cost of 1 apple is £2.50 and the cost of 1 chocolate bar is £1.50. Calculate the total cost of 4 apples and 6 chocolate bars.



Begin by looking at this question where it is more straight forward to find out the unknown (4 apples and 6 chocolate bars)

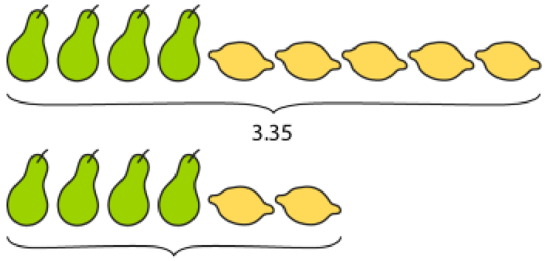
a=£2.50	a=£2.50	a=£2.50	a=£2.50	c=£1.50	c=£1.50	c=£1.50	c=£1.50	c=£1.50	c=£1.50
£19									

We already know what the value of each individual item was worth so we just needed to multiply up.

Look at this problem from NCETM PD Materials

'4 pears and 5 lemons cost £3.35.
4 pears and 2 lemons cost £2.30.'

- Pictorial – one diagram for each piece of information



We need to establish what is the difference between those two lines of fruit.

The difference physically is 3 lemons, and the difference numerically is £1.05. So now we know that 3 lemons cost £1.05. we can divide £1.05 by 3 and work out the value of one lemon.

£1.05		
35p	35p	35p

If we now know that each lemon is 35p we can put that information back into each bar model.

p	p	p	p	l	l	l	l	l
				35p	35p	35p	35p	35p
				£1.75				
£3.35								

Now we need to work out how much 4 pears are to be able to work out what one pear is.

p	p	p	p	l	l	l	l	l
40p	40p	40p	40p	35p	35p	35p	35p	35p
£1.60				£1.75				
£3.35								

One lemon = 35p
One pear = 40p
We can check this by looking to see if we can make the total for the second line of fruit with our new fruit prices

p	p	p	p	l	l
40p	40p	40p	40p	35p	35p
2.30					

Children to tackle questions like these ones below from the Ready to Progress Guidance

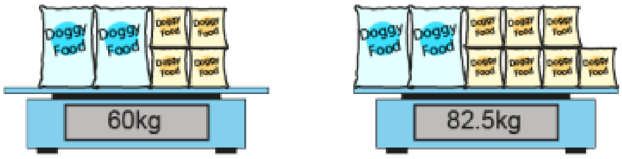
2. 1 eraser and 5 pencils cost a total of £3.35.

   5 erasers and 5 pencils cost a total of £4.75.

   a. How much does 1 eraser cost?

   b. How much does 1 pencil cost?

4. The balances show the combined masses of some large bags of dog food and some small bags of dog food.

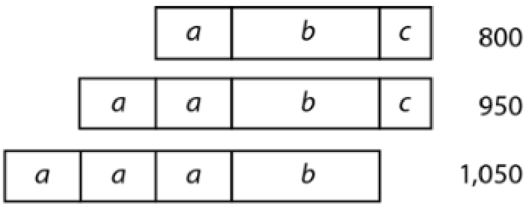


   How much does each bag-size cost?

Can children extend to similar questions that have been expressed algebraically like the one below from the NCETM PD Materials.

Can children see that the difference between the first and second row is one a on the lefthand side and 150 on the righthand side?

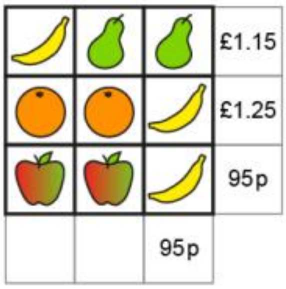
$$a + b + c = 800$$
$$2a + b + c = 950$$
$$3a + b = 1{,}050$$



$a = 150 \qquad b = 600 \qquad c = 50$

The question below adds further challenge for the children as they haven't got one row or column containing 3 of the same item.

6. The diagram shows the total cost of the items in each row and column. Fill in the 2 missing costs.



Children can use the top row and right-hand column as a starting point. There is a pear and banana in each group so at this stage we can represent them in their own bar model.

P and B
P and B

The top row as an additional pear and is 20p more expensive than the right column which has an additional banana. We can deduce that a pear is 20p more expensive than a banana.

P and B	P
P and B	B

20p

If we now look at the right-hand column of 2b + p = 95p, we can exchange a pear for a banana and take 20p off the total making 3b = 75p therefore b = 25p.

Solve problems with 2 unknowns and express this algebraically

Children now need to find the total of 2 unknowns. Use problems similar to those the children will have experienced in algebra and proportion to build on prior knowledge.

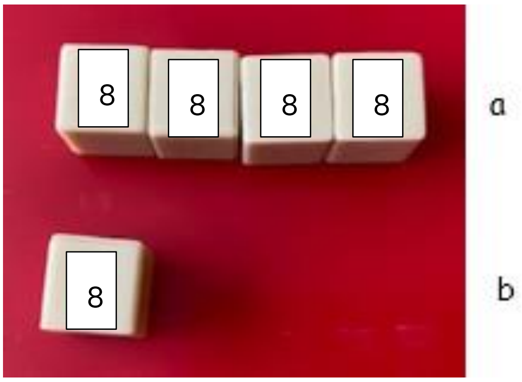
a and b total 40 but a is 4 times larger than b.

We could use a bar model or Cuisenaire to visualise this problem.

a

b

They have 5 parts. 40 divided by the 5 parts will equal 8 per part.

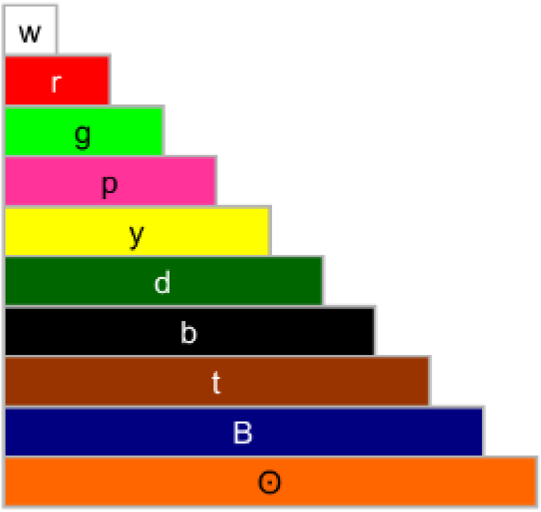


That would make a = 32 and b = 8
This fits with the original criteria because 32 is 4 times larger than 8.

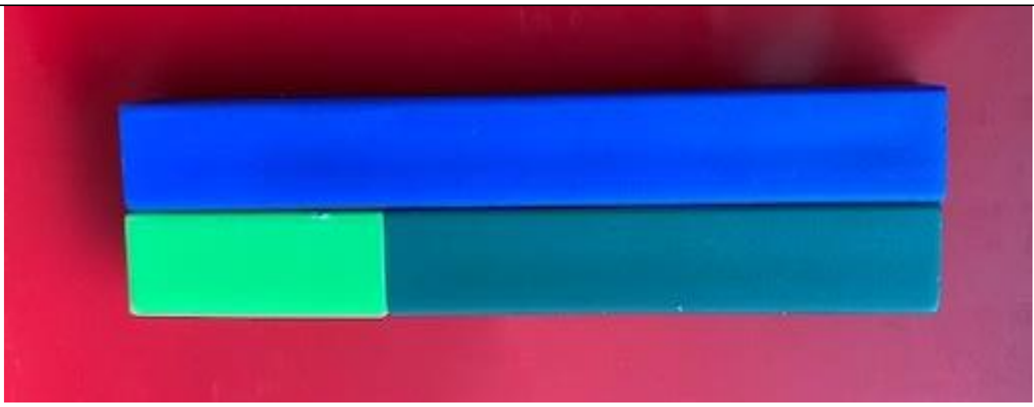
Ask children to tackle this problem using the same thinking.
a and b = 48.

a is one fifth the size of b. What is the value of each number?

Ask children to investigate with Cuisenaire



Tell the children to take the blue rod. Explain to them that 2 rods are the same length as the blue one and one of them is the dark green one. What is the other one? Allow children to investigate.



Demonstrate how to express this algebraically
B = g + d

Now explain to children that 2 rods total the blue rod but both of those rods are unknown. Can they now find more than one possibility?

Ask children to write these algebraically

B = b + r
B= t + w
B = p + y

Now ask children to consider additional factors when finding specific unknown combinations of rods to satisfy a specific criteria.

2 rods are the same length as one light green rod. One of the unknown rods is twice the size of the other.

Allow children time to investigate this and prove how they know that they have fulfilled the 'twice the size of criteria'.

g = r + w
R ÷ 2 = w
W x 2 = r (explain to children that we can also express this as r = 2w)

Now look at this example from the NCETM PD Materials

> *'I am thinking of two rods that together are equivalent to blue. There is a difference of white between the two rods. What are they?'*

B = p + y
p + w = y
y – w = p
Y – p = w

Ask children to do some investigating of their own with the Cuisenaire. Can they find combinations of rods that fulfil each of these stem sentences

__________ = ____________ and _______________

___________= white and _________________

_____________= _________________and _________________ but

______________ is double ______________

______________ = ____________ and _______________ with a

difference of ____________________

This problem is from a past Level 6 SATs paper but some children may be able to explore a solution using a bar model.

4 Here are some equations.

$X + Y = 50$,

$Y + Z = 70$ and

$X + Z = 80$

Find the value of **Y**.

2 marks

X = 30
Y = 20
Z = 50

Finding 2 unknowns in problems with different structures

Explain to children that in the Cuisenaire activity we didn't know the colours of the 2 unknown rods. We only know the criteria that they had to fulfil. We can relate the same thinking to mathematical problems

Explore this question from the ready to progress guidance

3. An adult ticket for the zoo costs £2 more than a child ticket. I spend a total of £33 buying 3 adult and 2 child tickets.
   a. How much does an adult ticket cost?
   b. How much does a child ticket cost?

We know the total cost, we know how many of each ticket was sold, we know that the children's tickets were £2 each less than the adults' tickets.

What we don't know are the 2 unknowns
The cost of each child's ticket
The cost of each adult's ticket
Show the children how to use a bar model to elicit what we know.

We know the whole is

£33

We know we need 5 parts for 5 tickets

c	c	a	a	a

We know that before we can work out each of the 5 parts, we need to consider that the children's tickets are £2 cheaper each which in turn means that the adults need to pay £2 more each. Let's take the £6 extra that the adults need to pay away from the whole for now and add it back on to the adult's tickets later.

£27				
5.40	5.40	5.40	5.40	5.40

We need to add on the £2 extra to each adults' ticket now to make sure that we account for the £6 that we took off the whole at the beginning.
Children's tickets cost £5.40
Adult's tickets cost £7.40

Work through this question from the NCETM PD materials

'Year 6 have earnt 200 stars; the stars are either gold or silver. They have 30 more gold stars than silver. How many are gold?'

Let's consider what the 2 unknowns are
Number of gold stars
Number of silver stars
We also know that the gold starts are 30 more than the silver stars

Our whole is 200
Gold + 30 = silver are the 2 parts

We need to think about taking the difference of 30 away from the whole to be able to find the 2 equal parts and to then add it back onto the gold total at the end.

Whole = 200		
30	170	
30	85	85
Gold = 115		Silver = 85

Ask children to tackle problems like this one

P and q = 1000
P is 150 greater than q
What is the value of p and q?

Generate and describe linear number sequences

Give children simple numerical sequences and ask them to explain what the rule is for each number sequence

3, 6, 9, 12
Rule – increasing by 3

9, 5, 1, -3
Rule – deceasing by 4

Children to look at sequences similar to the test questions below.

Discuss the inverse operations that are needed to solve the boxes to the left of the sequence.

The numbers in this sequence **increase** by 45 each time.

Write the missing numbers.

☐ 155 200 245 ☐ ☐

8 Here is a sequence of numbers starting from 220.

Each time 130 is subtracted from the number.

Write the **missing** numbers in the sequence given below.

220 ☐ ☐ -170 ☐

2 marks

In this sequence, the rule to get the next number is

Multiply by 2, and then add 3

Write the missing numbers.

	25	53	

10 In this sequence, the rule to get the next number is Multiply to 4, and then add 7.

Write the **missing** numbers

☐ 35 147 ☐

2 marks

21 The numbers in this sequence increase by 20 each time.

4 24 44...

The sequence continues in the same way.

Write the **two** numbers from the sequence that add to make a total of **108.**

☐ and ☐

1 mark

1 The list below shows the years in which the Football World Cup as held since 1982:

1982 1987 1991 1996 2001 2006 2011

George says,

George is **not** correct.

Explain how do you know.

Add in some sequences where children may need to think a bit harder about the sequence

4, 5, 7, 10, 14
Rule – the number that the value increases by goes up by 1 each time

5 The numbers in this sequence are first multiplied by 2 and then added by 3 each time.

Write the **missing** numbers.

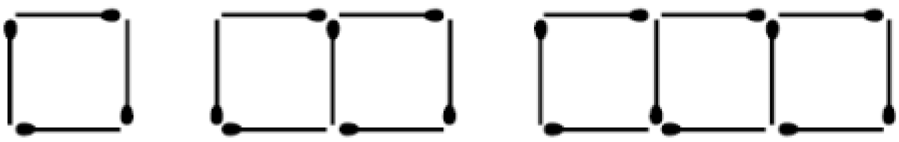
23 ☐ 101 ☐ 413

1 mark

Include the Fibonacci sequence and introduce the significance of this sequence to the children

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …

Mastery
Ramesh is exploring two sequence-generating rules. Rule A is: 'Start at 2, and then add on 5, and another 5, and another 5, and so on.' Rule B is: 'Write out the numbers that are in the five times table, and then subtract 2 from each number.' What's the same and what's different about the sequences generated by these two rules?

	Roshni and Darren are using sequence-generating rules. Roshni's rule is: 'Start at 4, and then add on 5, and another 5, and another 5, and so on.' Darren's rule is: 'Write out the numbers that are multiples of 5, starting with 5, and then subtract 1 from each number.' Roshni and Darren notice that the first few numbers in the sequences generated by each of their rules are the same. They think that all the numbers in the sequences generated by each of their rules will be the same. Do you agree? Explain your decision. **Mastery with Greater Depth** Ramesh is exploring three sequence-generating rules. Rule A is: 'Start at 30, and then add on 7, and another 7, and another 7, and so on.' Rule B is: 'Write out the numbers that are in the seven times table, and then add 2 to each number.' Rule C is: 'Start at 51, and then add on 4, and another 4, and another 4, and so on.' What's the same and what's different about the sequences generated by these three rules? Explain why any common patterns occur. Roshni and Darren are using sequence-generating rules. Roshni's rule is: 'Start at 5, and then add on 9, and another 9, and another 9, and so on.' Darren's rule is: 'Write out the numbers that are multiples of 3, starting with 3, and then subtract 1 from each number.' What might Roshni and Darren notice about the numbers in the sequences generated by each of these rules? Explain your reasoning.
nth term and formula for sequences	Ask children to build this pattern with match sticks  How many matchsticks are needed for the first part of the pattern (1st term)? How many for the second pattern (2nd term)? What if children continued the pattern on for a 4th time – how many matchsticks would be needed now? Ask children to complete a table showing the information from the pattern that they have already created.

Term	Matchsticks
1	4
2	7
3	10
4	13

What do the children notice about the number of matchsticks? How many matchsticks are we adding every time we want a new term? We are adding another 3 matchsticks.

Using this can they state how many matchsticks the 5^{th} term would have? Ask children what if we wanted to know how many matchsticks the 10^{th} term had or even the 100^{th}? It would take us a long time to add 3 every time.

Is there another pattern that we can find in the table above? Look at the numbers in the term section compared to the number next to it in the number of matchsticks row.

How can we get from 1 to 4?
How can we get from 2 to 7?
How can we get to 3 to 10?

Explain to children that what we do to the first numbers we need to do the same thing to the second and third pairs of numbers.

If we multiplied 1 by 3 and added 1 = 4
If we did the same thing to the numbers in the other rows, then we would get the correct answer
2x3+1= 7
3x3+1=10
4x3+1=13

How can this help us to work out the number of matchsticks in the 10^{th} term?

We need to start with 10 because it is the number of the term we are interested in and times that by 3 and then add 1. We should use 31 matchsticks in the 10^{th} term.

Ask children to work out how many matchsticks would be needed for the 100^{th} term?

We can now work out how many matchsticks are needed for any of the pieces of the pattern. Explain to children that this is called the nth term.

n – is whatever term you are interested in and x3+1 is what we do to that nth pattern piece to work out the number of matchsticks, this can be written as 3n + 1.

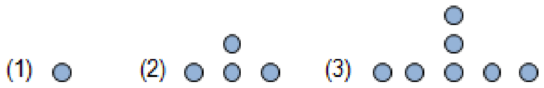
Can the children explore the patterns below in the same way?

generate and describe linear number sequences

Children should experience activities such as;

A number sequence is made from counters.

There are 7 counters in the third number.



How many counters in the 6th number? the 20th...?

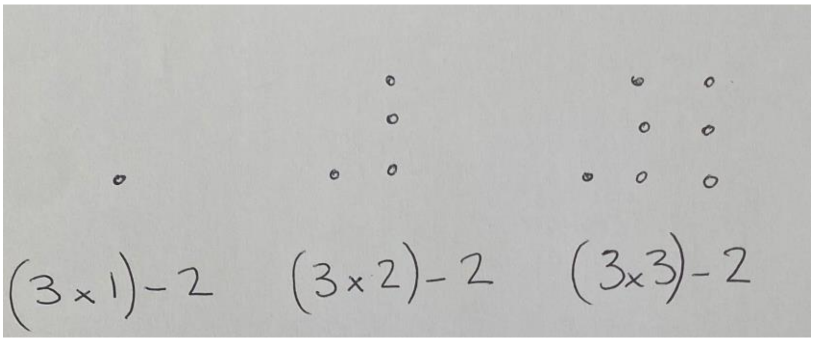
Write a formula for the number of counters in the nth number in the sequence.

Ask children to create a table for this pattern taken from the NCETM and to establish what the rule is for the nth term.

Term	1	2	3	4	5
Number in the sequence	1	4	7	10	13

This nth term is 3n – 2. Children may not spot this straight away, it may help to draw the terms as arrays to help spot the link between the 3 times table and the fact that 2 is always taken away from the array.

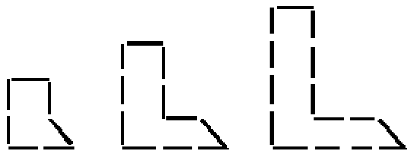
1st term 2nd term 3rd term



Now ask children to apply this to number sequences to establish any numerical value in the number sequence.

Apply to a range of SATs questions

Q1. Ann makes a pattern of **L** shapes with sticks.



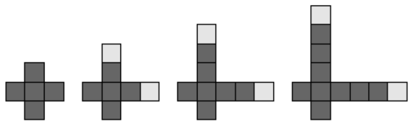
Shape–number:	**1**	**2**	**3**
Number of sticks:	7	11	15

Ann says : ***"I find the number of sticks for a shape by first multiplying the shape-number by 4, then adding 3".***

Work out the **number** of sticks for the shape that has shape-number **10.**

1 mark

Q6. Here is a sequence of shapes made from squares.



shape number (n)	1	2	3	4
number of squares (s)	5	7	9	11

There is a formula which connects the **number of squares (s)** used in a shape, with the **shape number (n)**.

Put a tick (✓) in the correct box.

$s = 3n + 2$ ☐

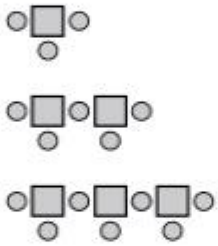
$s = 3n + 1$ ☐

$s = 2n + 3$ ☐

$s = 5n - 3$ ☐

$s = 2n + 5$ ☐

23 Here is a sequence of patterns made from squares and circles.



number of squares	number of circles
1	3
2	5
3	7

The sequence continues in the same way.

Calculate how many **squares** there will be in the pattern which has **25 circles**.

23

2 mark

True or false

'Sam says that the rule for the nth term for this pattern is 4n + 2 is he correct? How do you know?'

7, 11, 15, 19